Energy aware unicast geographic routing

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***Abstract*— In this paper, we are investigating the optimal radio range minimizing the energy globally consummed by a geographical routing process. Considering a geographical greedy routing protocol and a uniform distribution of nodes in the network area, we analytically evaluate the energy cost of a multi-hop communication. This cost evaluation corresponds to the asymptotic behavior of the routing protocol and turns out to be very accurate compared to the results obtained by simulations. We show that this cost is function of the node intensity and we use this result to deduce the optimal radio range. We evaluate this range with two energy consumption models, the first one considering the energy consumed by transmission operations only and the second one considering both transmission and reception operations. These results can be used in two ways. First, the nodes range can be tuned in advance as a function of the expected node intensity during an off-line planning. Second, we propose an adaptative algorithm where nodes tune their powers according to an on-line evaluation of the local node intensity.**

I. INTRODUCTION

A wireless ad hoc network is formed by a group of wireless hosts without any infrastructure. To enable communication, hosts cooperate among themselves to forward packets on behalf of each others. In particular, sensor networks are composed of low-power sensing devices used to collect data in a given area or to monitor and track remote objects. As they are equipped with a battery, energy is a scarce resource which limits the life of the network. Therefore, the stack of protocols used to maintain the network, collect and broadcast messages have to be chosen in order to minimize the consumed energy.

Classical ad hoc routing protocols are usually unadapted to sensor networks. For example, they are responsible for the dissemination of control packets which is not negligible in term of energy consumption. They generally involve the storage of topology informations and periodical computations of paths which are a serious drawback in the case of sensor nodes with low memory and CPU capacities; especially, in the case of large networks composed of several thousands of nodes such as microsensor networks [2].

In opposition to the energy consuming ad hoc routing protocols, stateless protocols, and in particular geographical ones, are generally envisaged. Under the assumption of some nodes relative positions knowledge, geographical routing protocols are very efficient regarding the different resources constraints: memory as the protocol is stateless, energy as the protocol does not disseminate many informations and computation as the choice of a next hop is mainly greedy and based on local geographic informations. For example in GPSR [3] (*Greedy Perimeter Stateless Routing*) of MFR [6] (*Most Forward Progress*), the closest destination neighbor is chosen as next hop.

A. Contributions

In this paper, we are investigating the optimal radio range minimizing the energy globally consumed by a geographical routing process. It has been shown in [4] that such a global optimal radius exists for specific contexts, when a regular pattern of nodes is assumed, and that this optimal radius depends on the considered routing mechanisms. In [7], authors study the optimal mean size of a hop given a realistic MAC layer. The MAC layer consists in emitting a packet at a given slot with probability p independently of other nodes and previous transmissions. The probability of transmission success is then a function of the distance between the transmitter and the receiver and the sum of the interferences generated by all other simultaneous transmissions. The quantity which is maximized is the product of the probability that the transmission succeeds and the hop size. If the hop size increases, the probability of success decreases as more transmissions will interfere. Authors deduce an optimal value for the hop size and for the parameter p. The approach is closed to what is done in out paper. Authors try, at each hop, to maximize the progress to the destination or equivalently to minimize the number of hops for the path between the source and the destination. In their case, they look for a tradeoff between the hop size and the generated interferences; in our case we look for a tradeoff between the hop size and the energy consumption.

Considering a geographical greedy routing protocol and a uniform distribution of nodes in the network area, we analytically evaluate the energy cost of a multi-hop communication. This cost evaluation corresponds to the asymptotic behavior of the routing protocol and turns out to be very accurate compared to the results obtained by simulations. We show that this cost is function of the node intensity and we use this result to deduce the optimal radio range. We evaluate this range with two energy consumption models, the first one considering the energy consumed by transmission operations only and the second one considering both transmission and reception operations.

These novel results can be used in two ways. First, the range of the nodes can be tuned in advance as a function of the expected intensity of nodes during an off-line design and provisioning. Second, we propose a distributed adaptive algorithm where nodes tune their powers according to an on-line evaluation of the local node intensity.

The paper is organized as follows, in Section II and Section III we present the model and the geographic routing protocol we consider for both analytical studies and simulations. In Section IV, we briefly present the computation of the cost function. In Section V, we study the cost of the energy consumed by transmitters and the related optimal

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radius when nodes are distributed in the plane. In Section VI, we extend these results to consider the energy cost induced by reception operations. Conclusions and future works are given in Section VIII.

II. Models

We model a sensor network by a point process distributed in $\mathbb{R}^2$. We consider a homogeneous Poisson point process Φ of intensity λ.

We consider a source S located at the origin and a destination D such that $|S, D| = d$. Two nodes x and y can communicate if and only if $d(x, y) < R_{max}$ with R_{max} being the maximum radio range that a node can achieve. Finally, we focus on the MFR (*Most Forward Routing* [6]) routing algorithm where each node on a path selects as next hop the node the closest to the destination within its radio range (R_{max}).

We evaluate the cost of a multi-hop communication between S and D as

$$\mathbb{C}ost = \mathbb{E}[\sum_{i=1}^{N} r_i^{\alpha} + \sum_{i=1}^{N} c]$$

where N is the number of hops from S to D, r_i is the size of the i^{th} hops, α is the distance-gradient value , c is a constant cost corresponding to signal processing. We consider the mean value of this cost. This corresponds to a classical expression of the energy spent during a transmission [4], [5].

We note that without the constant in the energy cost ($c = 0$), the optimal radio range which minimizes the cost function is nill. In this case, the optimal algorithm consists in choosing as next hop the closest node of the current node but which is closer to the destination than the current node. This routing algorithm has been recently studied in [8].

III. Problem statement

Considering the MFR routing algorithm, the path from the source to the destination may be represented by a Markov chain. Unfortunately, even if the routing decision is local, the length and the location of the next hop depend on the distance between the current node and the destination. In order to release from this dependency, we choose to consider the asymptotic behavior of the hop lengths as $d = |S, D|$ tends to infinity. With this assumption, the progress made by one hop is no more a function of the distance to the destination. We use this hop length to approximate the cost function. The cost is then supposed to be linear with the hop distance:

$$\mathbb{C}ost = \mathbb{E}[N]\mathbb{E}[r^{\alpha}] + \mathbb{E}[N]c$$

We evaluate $\mathbb{E}[N]$ as $\frac{d}{\mathbb{E}[a]}$ where a is the progress of a hop on the line (S, D) joining the source and the destination. Figure 1 illustrates the different notations. As we suppose d tending to infinity, we have $a = r \cos\theta$. The cost function is finally:

$$\mathbb{C}ost = \frac{d}{\mathbb{E}[r\cos\theta]} \left(\mathbb{E}[r^{\alpha}] + c\right) \quad (1)$$

This approximation has been shown to be very accurate by simulations. In the computation of $\mathbb{E}[a]$ and $\mathbb{E}[r^{\alpha}]$ we do not take into account the correlation between two successive hops.

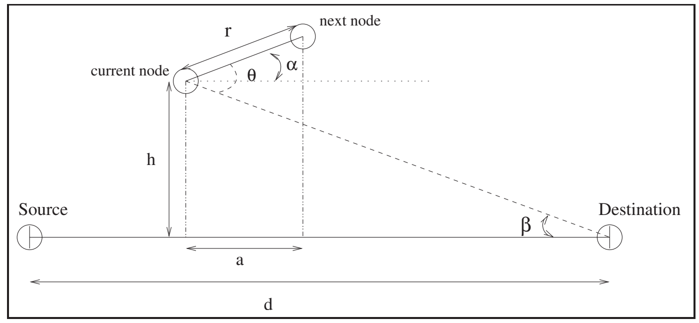

Fig. 1. Notation of the random variables used in the cost function.

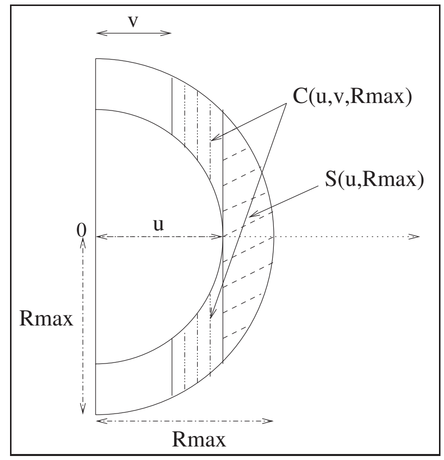

Fig. 2. The different notations

The distribution of the hop lengths depends on the location of the previous hop due to the Markovian nature of the routing algorithm. This dependency is due to the fact that the selection of a hop induces a region without any node. As simulations will show, neglecting this correlation has a very small impact on our estimation of the energy cost. Indeed, the next hop is chosen the closest to the destination and thus, in most cases, far from the current node. This limits the effect of the empty interval.

All these assumptions have a weak impact on the real cost as it will be shown later in this paper where simulations and analytical results are compared.

IV. Analytical computation of the cost function

In this Section, we briefly present the computation of $\mathbb{E}[r^{\alpha}]$ and $\mathbb{E}[r\cos\theta]$ which allows us to express the cost given by equation 1.

a) Computation of $\mathbb{E}[r^{\alpha}]$*:* In a first time we compute the distribution of the random variable r. More precisely, we compute $\mathbb{P}(r \leq u)$ for $u \in [0, R_{max}]$ in order to obtain the density function of r.

Let be $u \in [0, R_{max}]$. If there is at least one point in $S(u, R_{max})$ as represented in Figure 2 then $r > u$. We have,

$$\mathbb{P}(r \leq u) = \mathbb{P}(r \leq u|\Phi(S(u, R_{max})) = 0)$$
$$\times \exp\{-\lambda|S(u, R_{max})|\}$$

We compute $\mathbb{P}\left(r \le u | \Phi(S(u, R_{max})) = 0\right)$. It is equilavent to compute the probability that the first point reached by $S(v, R_{max})$ when v increases (from u to R_{max}) is in the half-circle of radius u denoted B_u rather than in the two bands denoted $C(u, 0, R_{max})$ as represented in the Figure 2.

In order to compute this probability, we project the points on the horizontal axe (represented in Figure 2) to obtain two point processes on the line. The projection of the points in $C(u, 0, R_{max})$ leads to the point process N_C and the projection of the points of the half-circlee B_u leads to the point process N_B. We have then two inhomogeneous Poisson point processes. With this notations, $\{r \le u | \Phi(S(u, R_{max})) = 0\}$ if and only if the closest point to 0 (in Figure 2) belongs to N_B rather than N_C. From the intensity measures of the two point processes, we obtain $\mathbb{P}(r \le u | \Phi(S(u, R_{max})) = 0)$ and after simple but heavy computation, we obtain:

$$\mathbb{P}\left(r \le u\right) = \lambda \int_0^u \frac{\partial}{\partial x} |S(u - x, u)|$$
$$\times \exp\{-\lambda |S(u - x, R_{max})|\} dx + \exp\{\frac{\lambda \pi R_{max}^2}{2}\}$$

The last exponentiel corresponds to the probability that there is no node in the half-circle of radius R_{max}. In this case $r = 0$. The distribution of r consists of an atom of size $\exp\{-\frac{\lambda \pi R_{max}^2}{2}\}$ at 0, and a component distributed on $(0, R_{max})$ with density:

$$f_r(u) = 4\lambda u \int_0^1 \sqrt{1 - y^2} \exp\{-\lambda R_{max}^2 T(\frac{uy}{R_{max}})\}$$
$$\times \left(1 + \lambda u y \sqrt{R_{max}^2 - u^2 y^2}\right) dy$$

and

$$\mathbb{E}[r^\alpha] = \int_0^{R_{max}} u^\alpha f_r(u) du$$

b) Computation of $\mathbb{E}[r\cos(\theta)]$*:* The computation of $\mathbb{E}[r\cos(\theta)]$ is easier. $\{r\cos(\theta) > u\}$ if and only if there is at least a point in the area $S(u, R_{max})$:

$$\mathbb{P}\left(r\cos(\theta) > u\right) = 1 - \exp\{-\lambda |S(u, R_{max})|\}$$

and

$$\mathbb{E}\left[r\cos(\theta)\right] = \int_0^{R_{max}} \left(1 - \exp\{-\lambda |S(u, R_{max})\}|\right) du$$

V. Evaluation

In this Section, we analyze the impact of the radius on the energy cost. We evaluate the energy cost using the function given by equation 1.

More precisely, we compare the proposed analytical cost value with cost results obtained by simulations. We then compare our analytical optimal radius with other results found in the litterature. We discuss the benefit of using our analytical optimal radius and compare our results to the least costly path. This path is found using the Dikjstra algorithm. In the last Subsection, we propose a distributed adaptative algorithm which allows a node to evaluate its local intensity and to deduce the radio scope it should use to minimize the global energy consumption.

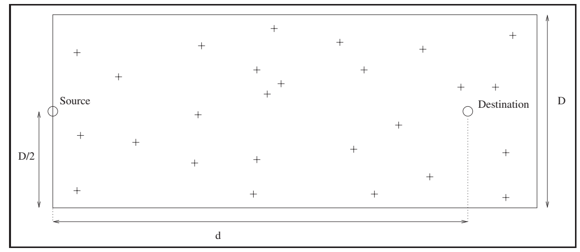

Fig. 3. Observation window used for the simulations.

c) Simulation model.: We consider a square of size $D \times 1.1d$ with $D = 10R_{max}$. The source and the destination are located at $(0, \frac{D}{2})$ and $(d, \frac{D}{2})$ as illustrated in Figure 3. The factor 1.1 is to handle the case where the last path forwarder is on the right of the destination. In this window, a homogeneous Poisson point process of intensity λ is generated. Routing is performed according to MFR.

The measured quantities are considered only when there is a path between the source and the destination. Therefore, we only consider process intensities λ for which a path exists with a no negligible probability. The different quantities are evaluated as the mean of 2000 samples.

A. Analytical results versus simulations

d) The cost function: In Figure 4, we compare the energy cost obtained analytically with the energy cost obtained by simulations. We make R_{max} vary from 50 to 100. It appears that our analytical cost function is very accurate and as a consequence, our analytical optimal radio range gets very close to the real optimal one.

In this Figure, we also compare the energy spent with our optimal radio range to the global minimum. In order to find this global minimum we consider that the radio range of nodes is not limited. Therefore, there is a link between all possible pairs of nodes. To each link, we associate an energy cost ($r^\alpha + c$ where r is the length of the link) equal to the energy needed to transmit on the link. Finally, we apply the Dijkstra algorithm to find the path which minimizes the consumed energy. For the considered intensity, it appears that the paths obtained with the geographical algorithm are very close to the global optimal paths. The difference is less than 3%. It is interesting to note that using a statefull routing protocol which allows the node to use the best path in terms of energy consumption will only reduce the cost by less than 3%. The complexity brought by the routing protocol (broadcast of control packets, routing table maintenance, computation of the shortest paths, etc.) is not worth such a little improvement.

B. Impact of the assumptions on the accuracy of the obtained optimal

e) Impact of d*:* The result of the previous paragraph is obtained with $d = 2000$. We have assumed that the cost function was proportional to d. As explained earlier, it is true for large d whereas small values of d can interfere with the

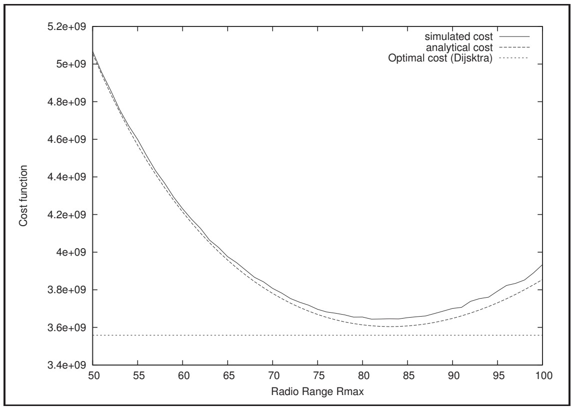

Fig. 4. Comparison of the cost function obtained by simulations with the cost function obtained analytically as R_{max} varies from 50 to 100 meters. $\lambda = 0.002547$, mean number of neighbors varying from 20 to 80 as R_{max} varies from 50 to 100, $d = 2000$

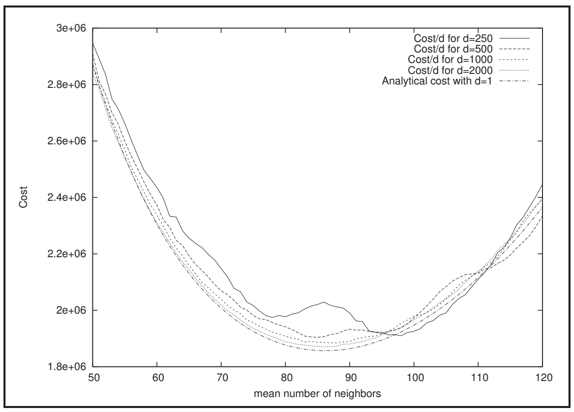

Fig. 5. Comparison of the normalized cost function obtained by simulations with the cost function obtained analytically as R_{max} varies from 50 to 100 meters. $\lambda = 0.001273$, mean number of neighbors varying from 10 to 40 as R_{max} varies from 50 to 100 meters.

assumed stationarity of the hops size. In order to evaluate the impact of d on the cost function, we compare $\frac{\mathbb{C}ost}{d}$ obtained analytically and the cost obtained by simulation. In Figure 5, we plot this normalized cost function for several values of d ($d = 250$, 500, 1000 and 2000). We observe that when $d = 250$ and 500, these costs fluctuate for certain values of R_{max}. However, it appears that even when d is small, the optimal found with our analytical cost function is still very accurate.

f) Mean cost versus sample cost: As the mean cost is not equal to the cost of all the samples, the optimal radius deduced from the mean cost may be far for some samples and thus may lead to poor performances for these samples. Indeed, for some statistical distributions, the mean value of the random variable can be really different from the observed samples. Power law distribution for instance shows such a behavior: there are a lot of observations with small values and sometimes a very high value. It leads to an average which does not appear in the samples. In Figure 6, we plot the mean cost (the same as in Figure 4), the minimal value among all the samples and the maximal value among all the samples. More specifically, for each R_{max}, we compute the cost for

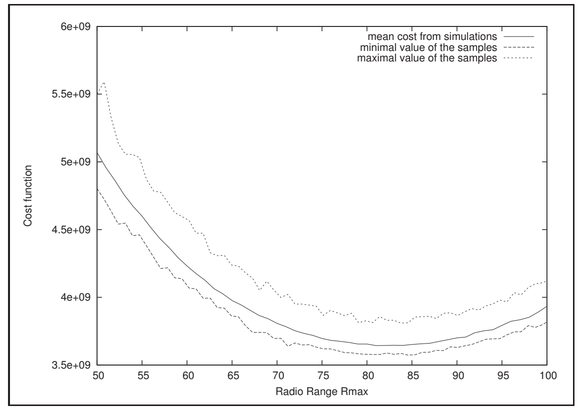

Fig. 6. Comparison of the mean cost function obtained by simulations with the lowest and highest values of the samples. $\lambda = 0.002547$, mean number of neighbors varying from 20 to 80 as R_{max} varies from 50 to 100 meters.

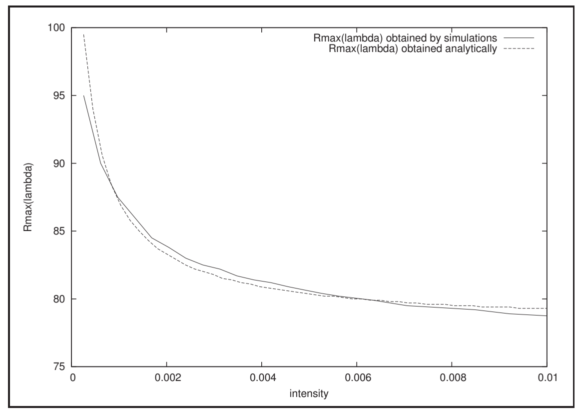

Fig. 7. The optimal radius $R_{opt}(\lambda)$ obtained from the simulations and from the analytical formula.

1000 samples and we keep the min and max costs from these 1000 values. The curves obtained from the 1000 samples are thus bound by these "min" and "max" curves. It appears that the minimal and maximal values are not far from the mean value and that the optimal radius does not change. Therefore, the optimal radio range deduced from the mean cost should offer very good performances for all the samples.

C. Optimal value of R_{max}

In Figure 7, we plot the optimal maximal radio range $R_{opt}(\lambda)$ obtained by minimization of the cost function described in 1.

It is interesting to note that when the intensity tends to infinity the optimal radius $R_{opt}(\lambda)$ tends to a constant $R_\infty = \left(\frac{c}{\alpha-1}\right)^{\frac{1}{\alpha}}$ for $\alpha > 1$. Indeed, when the intensity becomes really important, the path becomes very close to the line joining the source and the destination. The optimal radius R_∞ is then easily deduced from a model where nodes are distributed on the line.

D. An adaptive algorithm

Our study shows that the optimal radius is a function of the intensity of the nodes. However, this intensity is not always known in advance. In sensor networks, the intensity of the

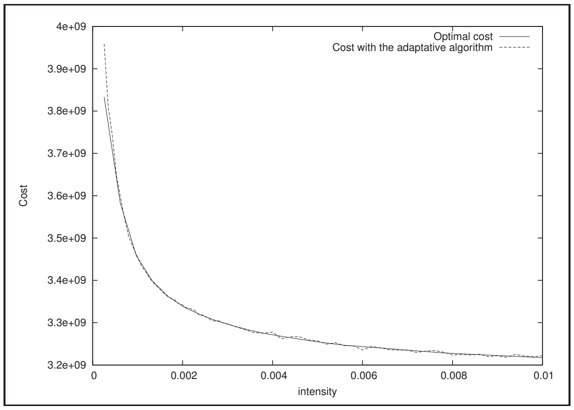

Fig. 8. Cost function with the adaptive algorithm.

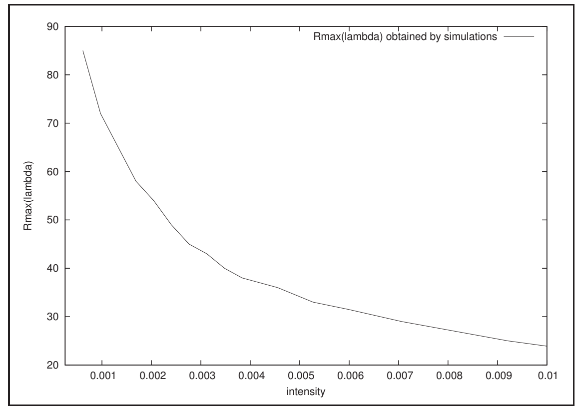

Fig. 9. The optimal radius when the receptions are taken into account.

nodes may be fixed in order to guarantee a certain coverage of the sensored area. Thus, for such networks, radio range of the nodes may be tuned off-line in the sensors according to the intensity. In most of the others wireless multi hop wireless networks, the global intensity of the process is not known in advance. Therefore, we propose an adaptive algorithm which allows a node to choose its radio scope as a function of its neighborhood. We assume that each node has a certain initial radio range $Range$ and that each node knows the number of nodes in its radio range. The local intensity is then evaluated as :

$$\overline{\lambda} = \frac{|\text{number of nodes in the radio range}|}{\pi Range^2}$$

Each node then evaluates its optimal radius as a function of $\overline{\lambda}$: an array gives the radius w.r.t. the evaluated intensity. We simulate this adaptive algorithm to compare with the optimal cost obtained previously. We consider the same model of simulation as in the other Sections. Each node has an array of 22 values which maps the estimated intensity and the corresponding optimal radio range. This optimal radio range are the values obtained in Section V-C using our analytical study. In Figure 8, the cost between the adaptive algorithm and the optimal cost where the global intensity is known are compared. In this Figure, the initial range $Range$ is equal to 100. The adaptive algorithm gives very good results as its cost is the same as the optimal cost with the intensity being known.

VI. COST INCLUDING THE RECEPTIONS

In the previous Sections, only the cost induced by the transmissions has been taken into account. However, receptions also consume energy. As mentioned in the Introduction, for some technologies, receptions are in the same order of magnitude as transmissions in terms of energy consumption ([1]). If we add a cost γ for each reception, the global cost becomes:

$$\mathbb{C}ost = \mathbb{E}[\sum_{i=1}^{N} r_i^{\alpha} + \sum_{i=1}^{N} c + \gamma \sum_{i=1}^{N} \Phi(B_{x_i}(r_i))]$$

where $\Phi(B_{x_i}(r_i))$ is the number of receptions generated by the i^{th} transmissions (x_i is the location of the i^{th} transmitter).

In Figure 9, the optimal radius and the cost at the optimal are shown when the intensity of the process increases. It appears that the optimal radius is smaller than in the other cases. Contrarily to the case where receptions are not considered, the optimal radio range does not converge to a positive limit. As the intensity increases, the cost induced by receptions increases and the radius is reduced.

g) Discussion: When the receptions are not considered, the optimal cost converges as the intensity increases. The cost is then minimal when there is an infinite number of nodes. The path consists in a sequence of deterministic size of hops of size R_∞ on the line joining the source and the destination. In the case where receptions are taken into account, the most dense is the network, the highest is the cost of a transmission. The optimal radius decreases with the intensity increasing and tends to 0 as the intensity λ tends to infinity. The cost is smaller when there is a smaller number of nodes in the network as, in this case, a high radio range can be chosen, leading to a small number of receptions and a small number of hops to the destination.

VII. THE INHOMOGENEOUS CASE

Both analytical results and simulations hold for nodes distributed by a homogeneous point process. But, is there an optimal radius when nodes are distributed in an inhomogeneous way? And, does the adaptive algorithm still work? We cannot answer to these questions for general inhomogeneous point processes as it depends on the density function of these processes. Although, we can consider a particular inhomogeneous point process and show that, in this case, there is an optimal radius and that our adaptive algorithm gives very good results.

The point process is the following: we distribute a first homogeneous point process and superpose several point processes which model concentration areas. A sample of such a process is illustrated in Figure 10.

We compute the cost function (without the receptions) for two cases. The case where the radio range is fixed and the same for all the nodes. We vary the radio range from 50 to 120. The second case corresponds to the adaptive algorithm. The radio range is then a function of the number of neighbors (within an initial radio range $Range = 100$ meters in our simulations). The two curves are shown in Figure 11.

The cost with the adaptive algorithm is constant as the radio range is computed by each nodes. When the radio range is the

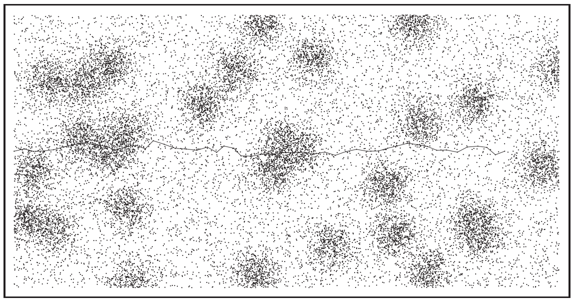

Fig. 10. A sample of the inhomogeneous point process.

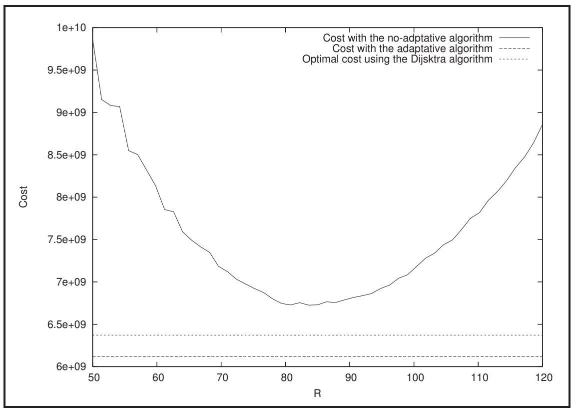

Fig. 11. The cost function when R varies and with the adaptive algorithm. $d = 4000$.

same for all the nodes, we observe an optimal value which is here approximately 85. But the most interesting result is that the adaptive algorithm gives better results in terms of energy consumption. We suppose that it is due to the fact that in the inhomogeneous case, the adaptive algorithm adapts to the particular concentration zones of the network. Each node adapts its radius according to its number of neighbors. The radius will be high when there are few nodes in the neighborhood and smaller in high concentration areas. In the case where R_{max} is fixed for all the nodes, the optimal radius found in Figure 11 is then a tradeoff between concentration zones and zones with a lower number of node. This tradeoff leads to a higher energy consumption.

We also consider the optimal cost among all the possible paths. It is obtained with the same algorithm as in Section V-D. We associate to each link a cost corresponding to the cost of a transmission on the link and we apply the Dijkstra algorithm on the full mesh. The difference between the global minimum obtained with the Dijkstra algorithm and our adaptive algorithm is less than 4%. Therefore, for the considered point process, the benefit of reactive/proactive routing protocols does not justify their use.

VIII. Conclusion

In this paper we have addressed the problem of evaluating the radius minimizing the energy consumption during unicast communications in MANET and sensor networks. We concentrate only on cases where nodes of the network are randomly distributed. We have showed that such an optimal radius exists and that it depends on the intensity of the process. We have proposed an analytical cost function which is showed to be close to the cost function obtained by simulations. This analytical function is then used in order to deduce the optimal radius in a more convenient way than using simulations. In this study, we consider the costs of both transmission and reception operations. We have proposed an adaptive algorithm which assumes that nodes can adapt their power in order to tune their radio range. It appears that this adaptive algorithm minimizes the cost function as well as the algorithm which uses the same fixed radius for all nodes. It also turns out that this algorithm is very efficient in a particular inhomogeneous process with high concentration zones. This result must be generalized in future works to general inhomogeneous processes.

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