Throughput Limits in Spectrum Sensing Cognitive Radio Networks using Point Processes Invited Paper

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Abstract—**Spectrum sensing in cognitive wireless networks is important for Secondary nodes to avoid interference with the Primaries. In this paper, considering a spectrum sensing framework, we develop models for bounding interference levels from Secondary network to the Primary nodes. First, we assume that both networks are distributed according to Poisson point processes. Subsequently, we consider a more practical model which takes into account the medium access control regulations and where the Secondary Poisson process is judiciously thinned in two phases to avoid interference with the Secondary as well as the Primary nodes. The resulting process will be a modified version of the Matèrn point process. For both of these models, we obtain bounds for CCDF of interference and present simulation results which show the developed analytical bounds are quite tight. Moreover, we use these bounds to find the operation regions of the Secondary network such that the interference constraint is satisfied on receiving Primary nodes for the two cases. We then obtain theoretical results on the Primary and Secondary throughput and find the throughput limits under interference constraint.**

I. Introduction

Dynamic spectrum access and management provides an opportunity to use the limited radio frequency more efficiently. This is irrefutably needed as there is a demand for higher transmission rates and increased network throughput. While this notion in general encompasses a variety of wireless systems, one important scenario of interest is the concept in which the unlicensed users are allowed to access the spectrum licensed to the incumbent users on a non-interfering or limited interference basis. The practical solution requires wireless devices with cognitive radio capability to share the bandwidth with Primary users.

Considerable research has been undertaken in the area of dynamic spectrum access and management and cognitive networks (see for example, [1], [2], and [3]). To implement such systems, various approaches have been discussed that involve issues ranging from spectrum opportunity identification and exploitation to Media Access Control (MAC) protocol. For example, reference [2] attempts to design a cognitive MAC protocol, and optimal and suboptimal solutions are proposed for dynamic spectrum access, assuming that users can partially observe the instantaneous spectrum availability and individually decide which channels to sense and access. One important aspect is spectrum sensing, which enables the Secondary nodes to be perceptive of the spectral activity of the Primary users and thereby avoid and manage their level of interference.

What gives rise to such concepts to become realistic is managing the level of interference being harmful to the incumbent users. Therefore, an understanding of the characteristics of interference and its behavior, which is stochastic, is at the core of the problem of determining the degree of bandwidth efficiency and hence useful capacity to be used by secondaries. This is precisely our focus here and we develop analytical models and bounds for the level of interference in order to determine the throughput. Published work in [4] considers interference modeling in spectrum underlay cognitive wireless networks and interference is approximated as sum of Normal and Log-normal random variables.

We consider Secondary nodes to monitor individual transmissions from Primary nodes. Upon detecting no activities, they can transmit. In this paper, using techniques from stochastic geometry and the theory of point processes, we develop models for bounding the CCDF (Complementary Cumulative Distribution Function) of interference level from Secondary nodes to a Primary node for Poisson point processes representing both Primary and Secondary nodes. Subsequently, we consider a more practical model which takes into account the medium access control regulations and where the Secondary Poisson process is judiciously thinned in two phases to avoid interference with the Secondary as well as the Primary nodes. The resulting process will be a modified version of the Matèrn point process. We model the CCDF of interference level from Secondary nodes to a Primary node for this Matèrn point process representing Secondary nodes. Throughput estimation for Primary and Secondary nodes are of interest. In [5], considering a simple Gaussian model, throughput in Primary and Secondary networks are optimized by using the optimum transmission probability. We use our obtained models to find the operation regions of the Secondary network such that the interference constraint is satisfied on receiving Primary nodes for the two cases. We then obtain theoretical results on the

Primary and Secondary throughput and find the throughput limits under interference constraint.

In the remainder of this paper, we discuss the model used and obtain analytical results for the bounds on CCDF of interference in section II. Numerical evaluations and simulations are also provided to confirm the accuracy of the obtained results. In section III, theoretical results are found for the throughput considering the two models used in this paper. Section IV, considers the throughput under the interference constraint and section V concludes the paper.

II. Model

We focus on the interference level at location O (the origin of the plane) and at a given time t. Interference is assumed to be the sum of signal strengths generated by all the nodes transmitting at time t. We use the following notations to denote interference from Primary (I_P) and Secondary to Primary networks ($I_{S\to P}$):

$$I_P = \sum_{i=1}^{+\infty} P_P \xi_i l\left(\|Y_i\|\right) \text{ and } I_{S\to P} = \sum_{i=1}^{+\infty} P_S \zeta_i l\left(\|X_i\|\right) \quad (1)$$

where $\{\zeta_i\}$ and $\{\xi_i\}$ are i.i.d. random variables representing fading, $l\left(\|.\|\right)$ represents deterministic path loss (a decreasing function), P_P and P_S are the transmission powers from Primaries and Secondaries, and $(Y_i)_{i\in I\!N}$ (resp. $(X_i)_{i\in I\!N}$) represent locations of the interfering nodes in the Primary (resp. in the Secondary network). We assume that fading is Rayleigh. Consequently, in the following we consider the random variables $\{\zeta_i\}$ and $\{\xi_i\}$ to be exponentially distributed with parameters equal to 1.

It is obvious that according to equations (1), transmitters' locations play a crucial role on interference. Interference distribution strongly depends on the distribution of the simultaneous transmitters, i.e., $(X_i)_{i\in I\!N}$ and $(Y_i)_{i\in I\!N}$ distributions. We consider two stationary point processes Φ_P ($\Phi_P = \{Y_i\}_{i\in I\!N}$) and Φ_S ($\Phi_S = (X_i)_{i\in I\!N}$) describing locations of the Primary and the Secondary nodes, respectively. Basically, a point process consists of a random sequence of points distributed in $I\!R^d$ (See [6] for a deeper presentation). In the two next sub-sections, we present the different point processes used to model transmitters' locations.

A. *Model 1: Poisson*

We consider Φ_P and Φ_S to be two Poisson point processes with intensity λ_P and λ_S, respectively. A sample of this model is presented in Figure 1(a).

B. *Model 2: a modified version of the Matèrn point process*

In this model, we assume that a Secondary node listens to the medium before transmitting. If it detects the transmission of a frame from another Secondary node or a Primary node, it delays its own transmission. We assume that a transmission is detected by a node if the received signal strength from another node is greater than a threshold γ. We also consider a simplified deterministic path loss and assume that the received signal strength is $Pl(u)$ where u is the distance between the two nodes and P is the transmission power. For a given value of γ, there is therefore a maximal distance for which a transmission is detected. As this distance depends on the transmission power, we consider two different thresholds. The distance h_P for the Primary nodes is the solution of $\gamma = P_P l(h_P)$, and h_S for the Secondary nodes is the solution of $\gamma = P_S l(h_S)$. The Matèrn point process is suitable to model the transmitters' positions when using this medium access protocol. Basically, it is formed by removing a subset of the points of a Poisson point process in such a way that distances between all the pair of remaining points are greater than a predefined constant (h_S or h_P in our case). This model has been already used to model such networks in [7] and [8]. We propose a modified version of the Matèrn point process in order to take into account detection from both Primary and Secondary nodes. We present below the classical Matèrn point process, followed by a modified version which suits the context of our problem.

a) Definition of Matèrn process: We consider a homogeneous Poisson point process Φ with intensity λ. We associate with each point x a random variable $m(x)$ independently and uniformly distributed in $[0, 1]$. We perform a dependent thinning of the Poisson process. We retain a point x if and only if the points in the ball $b(x, h)$ contains no points with marks smaller than $m(x)$. Formally, the points of the Matèrn is the set $\{x \in \Phi | m(x) < m(y) \forall y \in \Phi \cap b(x, h) \backslash x\}$. The intensity λ_h of this process is known (see for instance [6] page 164) and is given by:

$$\lambda_h = \frac{1 - \exp\left\{-\lambda \pi h^2\right\}}{\pi h^2} \quad (2)$$

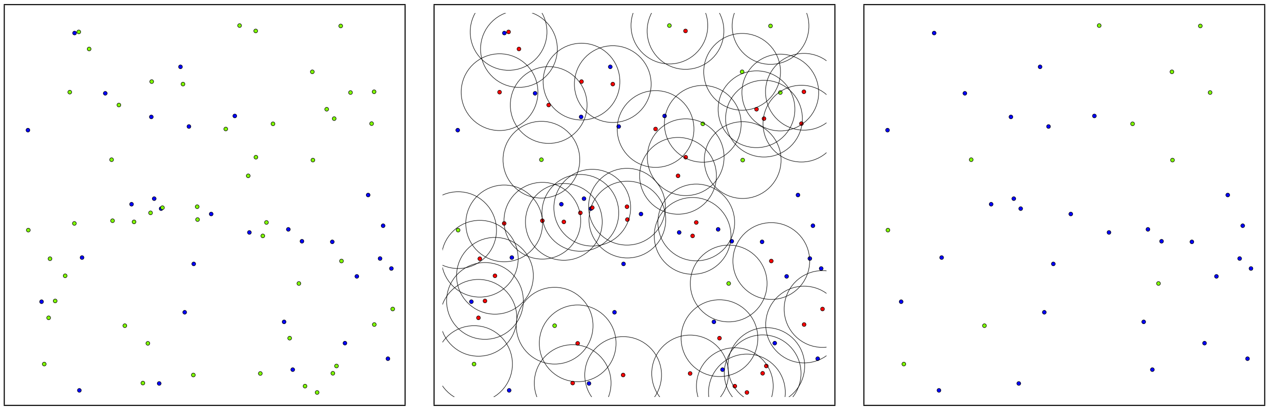
b) Our model: We use a modified version of the Matèrn point process as the Primary nodes do not apply the same rule to access the medium. The model is as follow:

- Simultaneous transmitters of the Primary network are distributed according to a Poisson point process Φ_P with intensity λ_P.
- All the Secondary nodes are distributed according to a Poisson point process Φ_S with intensity λ_S.
- We consider a classical Matérn point process with Φ_S as the underlying Poisson process and distance threshold h_S. It corresponds to a first thinning of Φ_S by taking into account transmission from secondary nodes.
- The Matèrn point process is thinned a second time to take into account the transmission from the Primaries. If a point of the Matèrn is located at a distance less than h_P from a Primary transmitter, it is removed.

The intensity of the selected Secondary nodes denoted by $\lambda_S^{'}$ is then given by:

$$\lambda_S^{'} \quad = \quad \exp\left\{-\lambda_P \pi h_P^2\right\} \frac{1 - \exp\left\{-\lambda_S \pi h_S^2\right\}}{\pi h_S^2} \quad (3)$$

It is the Matèrn's intensity multiplied by the probability for a Primary node of lying at distance less than h_P of a secondary.



(a) Model 1. Primaries (Blue) and Secondaries (Green) are Poisson.

(b) Same sample as Figure 1(a). Primary nodes are blue. Inhibition balls with radius h_S are plotted around the Secondary nodes. Secondary nodes which are going to be removed (due to the two successive thinning) are in red. The selected Secondary nodes are green.

(c) Model 2. We keep only those secondary nodes which do not have Primaries within their inhibition ball and satisfy the Matèrn condition on the marks.

Fig. 1. Model 1 and Model 2. For model 2, we start from model 1 where both Primaries and Secondaries are Poisson. Then, we remove a Secondary if it has a Primary in its ball, or if it does not have the highest mark compared to other secondary nodes within its ball. It is shown in Figure (b). The final point processes considered in model 2 are shown in Figure (c).

A sample of this model is presented in Figure 1(a). The way it is built from the two Poisson point processes is shown in Figures 1(b) and 1(c).

III. Complementary Cumulative Distribution Function of Interference

In cognitive radio networks, Secondary nodes must keep a low interference level in order to ensure that performance of the Primary network is not significantly deteriorated. Interference from Secondary to Primary must satisfy the following condition:

$$\mathbb{P}\left(I_{S\to P} > \eta\right) < \epsilon \tag{4}$$

For the two models, we develop bounds and approximation on this probability to determine the parameters for the Secondary network for which this condition is met. We consider a node receiving a frame from a Primary node. This node is assumed to be located at the origin of the plane.

A. Bound for Model 1 (Poisson)

First, we propose a lower bound on the Cumulative Distribution Function (cdf) of $I_{S\to P}$. We then deduce an upper bound on the Complementary Cumulative Distribution Function (CCDF). We also propose an approximation easier to compute than the bound. The interference cumulative distribution function is defined as $\mathbb{P}\left(I_{S\to P} \le \eta\right)$.

Proposition 1. *The lower bound is computed as follows:*

$$\mathbb{P}\left(I_{S\to P} \le \eta\right) \ge 1 - 2\pi\lambda_S \int_0^{+\infty} \exp\left\{-\frac{\eta}{P_S l(r)}\right\} \times \exp\Bigg\{ -\lambda_S 2\pi \int_r^{+\infty} (1 - \frac{1-\exp\left\{-\frac{\eta}{P_S}\left(\frac{1}{l(w)} - \frac{1}{l(r)}\right)\right\}}{1-\frac{l(w)}{l(r)}}) w dw \Bigg\} r dr \tag{5}$$

The upper bound on the complementary cumulative distribution function is then:

$$\mathbb{P}\left(I_{S\to P} \ge \eta\right) \le 2\pi\lambda_S \int_0^{+\infty} \exp\left\{-\frac{\eta}{P_S l(r)}\right\} \times \exp\Bigg\{ -\lambda_S 2\pi \int_r^{+\infty} \left(1 - \frac{1-\exp\left\{-\frac{\eta}{P_S}\left(\frac{1}{l(w)} - \frac{1}{l(r)}\right)\right\}}{1-\frac{l(w)}{l(r)}}\right) w dw \Bigg\} r dr \tag{6}$$

The proof is given in appendix. The approximation used to evaluate the CCDF is found by taking the second integral of Equation (6) equal to 0 (it is a good approximation as this integral is very small in practice):

$$\mathbb{P}\left(I_{S\to P} \ge \eta\right) \le 2\pi\lambda_S \int_0^{+\infty} \exp\left\{-\frac{\eta}{P_S l(r)}\right\} r dr \tag{7}$$

B. Bound for model 2 (modified Matèrn)

We consider the modified version of the Matèrn point process to model the Secondary nodes (presented in Section II-B). We compute interference for a node located at the origin of the plane O. This node receives a frame from a Primary node located at a distance d. Without loss of generality, we assume that this node is located at $T = (d, 0)$. Therefore, there is an inhibition ball $b(T, h_P)$ centered at T and with radius h_P around this transmitter. From the intensity of the modified

Matèrn (see equation (3)), it is easy to find an upper bound on the interference generated by the Secondary nodes. It is found by using the Markov inequality:

$$\mathbb{P}\left(I_{S\to P} > \eta\right) \quad \leq \quad \frac{\mathbb{E}\left[I_{S\to P}\right]}{\eta} \tag{8}$$

Since the modified Matèrn is stationary, we can apply Campbell formula (see [6] page 104) to compute mean interference (with $\lambda_S^{'}$ given by equation (3)):

$$\mathbb{E}\left[I_{S\to P}\right] \quad = \quad \lambda_S^{'} \int_{I\!R^2\setminus \overline{b(T,h_P)}} \mathbb{E}[\zeta] l\left(\|u\|\right) du \tag{9}$$

This bound may be hard to compute, therefore we propose instead an approximation to compute the CCDF. It has been shown through a statistical study of interference [10], that interference generated by a Matèrn point process follows a log-normal distribution. In order to determine the two parameters of this distribution, we use mean and variance of interference. The mean interference is given by formula (9). The second moment of interference generated by a Matèrn point process has been computed in [9]. We obtain a variant of this second moment for our model. Let us define $B_u = b(u, h_S)$ and $\nu(A)$ the Lebesgue measure of A (area of A) for $A \subset I\!R^2$. We have:

$$\begin{aligned}
&\mathbb{E}\left[I_{S\to P}^2\right] = \frac{1-\exp\left\{-\lambda_S \pi h_S\right\}}{\pi h_S^2} \exp\left\{-\lambda_P \pi h_P\right\} \\
&\times \int_{I\!R^2\setminus b(T,h_P)} P_S^2 \mathbb{E}\left[\zeta^2\right] l(\|x\|)^2 dx \\
&+ \frac{2P_S^2}{\pi h_S^2} \int_{I\!R^2\setminus b(T,h_P)} \int_{I\!R^2\setminus (B_x \cup b(T,h_P))} \mathbb{E}[\zeta_1\zeta_2] \\
&\times \mathbb{E}\left[\frac{1-\exp\left\{-\lambda_S \nu(B_x\cup B_y)\right\}}{\nu(B_x\cup B_y)}\right. \\
&- \left.\frac{\exp\left\{-\lambda_S \pi h_S^2\right\} - \exp\left\{-\lambda_S \nu(B_x \cup B_y)\right\}}{\nu(B_x\cup B_y) - \pi h_s^2}\right] \\
&\times \exp\left\{-\lambda_P \nu(b(x,h_P)\cup b(y,h_P))\right\} l\left(\|x\|\right) l\left(\|y\|\right) dy dx
\end{aligned} \tag{10}$$

with $\mathbb{E}[\zeta^2] = 2$ and $\mathbb{E}[\zeta_1\zeta_2] = 1$. The approximation is then $I_{S\to P} \rightsquigarrow logNormal(m, \sigma^2)$ where m and σ^2 correspond to mean and variance of this log-normal distribution, m is given by Equation (9) and $\sigma^2 = \mathbb{E}\left[I_{S\to P}^2\right] - m^2$ with $\mathbb{E}\left[I_{S\to P}^2\right]$ given by Equation (10).

C. Numerical evaluation and simulation

Figures 2(a) and 2(b) show the CCDF obtained by simulations, the upper bound for model 1 given in Proposition 1 and the two approximations using Equation (7) for model 1 and the log-normal distribution for model 2. These simulations are used to observe the accuracy of the bounds and to see the difference between our approximation and real values. The upper bound and the approximations are very close to the simulations. It is worth noting that we have repeated our simulations for different values of λ_S and they also lead to very comparable results which are not shown here due to lack of space. For the two models, the bound and the approximations can be used to determine the values of λ_S for which interference satisfies the constraint given by Equation (4). For instance, in model 1, λ_S can be seen as the result of a thinning of a Poisson point process($\lambda_S = p\lambda$), where λ_S is the intensity of the simultaneous transmitter and λ the intensity of all the secondary nodes. If we consider that the intensity of the Secondary nodes which want to transmit to be λ and the access protocol to be ALOHA, we can tune the parameter p to guarantee the condition on the CCDF. For model 2, we can use the log-normal approximation to determine values of h_S satisfying Equation (4). We can then deduce the threshold γ from the solution of $\gamma = P_S l(h_S)$, which should be used by the secondary nodes to detect a frame transmission.

IV. THROUGHPUT ESTIMATION

In this Section, we focus on the obtainable throughput by both primary and secondary networks. This throughput is defined as the mean number of frames that are correctly received per second in a unit square area. We estimate the throughput as follows:

$$throughput = \lambda(1 - FER)\frac{1}{T} \tag{11}$$

where λ is the intensity of the simultaneous transmitters, T is the mean time required to send a frame, and FER is the Frame Error Rate. We compute this quantity for the two models that we have developed. For both models, Primary nodes are distributed according to a Poisson point process. Secondary nodes are also Poisson in the first model (Section IV-A), and are distributed according to our modified Matèrn in the second model (Section IV-B).

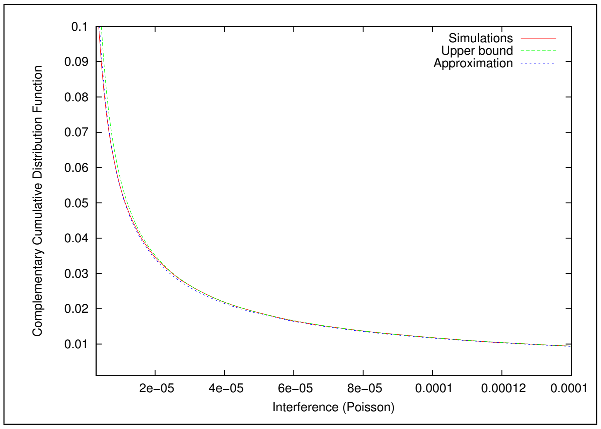
A. Model 1: Poisson

We consider the Frame Error Rate for a node which is located at the origin and is receiving a frame from a primary node at distance d. We use the definition and method developed in [7]. We use the following estimation for the Frame Error Rate:

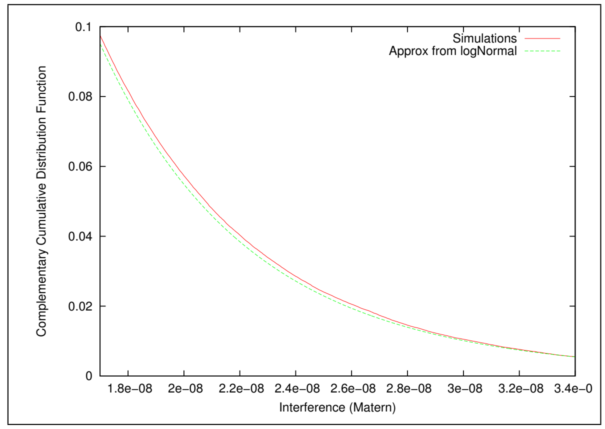
$$FER \quad = \quad \mathbb{P}\left(SIR < \theta\right) \tag{12}$$

where SIR is the ratio of the power received from the transmitter and the sum of the interference generated by the Primary and Secondary nodes. In the proposition below, we give the throughput for the Primary and Secondary networks. The proof is given in [11].

Proposition 2. *For model 1, throughputs of Primary and Secondary networks are:*



(a) Complementary Cumulative Distribution Function of $I_{S \to P}$ for the Poisson point process and the upper bound.



(b) Complementary Cumulative Distribution Function of $I_{S \to P}$ for the modified Matèrn point process and the approximation from log-normal.

Fig. 2. CCDF of $I_{S \to P}$ obtained by simulation. $I_{S \to P} = \sum_{i=1}^{+\infty} P_S \zeta_i l(\|X_i\|)$. $l(u) = min\left(\left(\frac{\beta}{4\pi u}\right)^{\alpha}, 1\right)$. $\beta = 0.346$ meters (wavelength). $\alpha = 3$. $\lambda_S = 0.001$. $P_S = 40$mW. $h_S = 50$ meters. $h_P = 50$ meters. $d = 0$. ζ_i is exponentially distributed with parameter 1. We considered $1,000,000$ samples (it is why we do not need to show the confidence interval as it is negligeable).

$$\begin{aligned} & throughput_{Primary} \\ = \quad & \lambda_P \exp\left\{-\lambda_S 2\pi \int_0^{+\infty} \frac{\theta P_S l(r)}{P_P l(d) + \theta P_S l(r)} r dr\right\} \\ \times \quad & \exp\left\{-\lambda_P 2\pi \int_0^{+\infty} \frac{\theta l(r)}{l(d) + \theta l(r)} r dr\right\} \frac{1}{T} \qquad (13) \end{aligned}$$

$$\begin{aligned} & throughput_{Secondary} \\ = \quad & \lambda_S \exp\left\{-\lambda_S 2\pi \int_0^{+\infty} \frac{\theta l(r)}{l(d) + \theta l(r)} r dr\right\} \\ \times \quad & \exp\left\{-\lambda_P 2\pi \int_0^{+\infty} \frac{\theta P_P l(r)}{P_S l(d) + \theta P_P l(r)} r dr\right\} \frac{1}{T} \qquad (14) \end{aligned}$$

The proof is given in appendix. It is worth noting that interference at the receiver is the same when this receiver belongs to the Primary or the Secondary network. When the receiver is a Primary node, located at the origin, we assume that there was a Primary transmitter at distance d from this receiver. As the considered point process, conditional on the presence of this transmitter, is Poisson, the other Primary transmitters are still Poisson with the same intensity (this result is given by the Slyvniack's theorem ; see [6] page 121). This transmitter is not taken into account in computation of interference, the interfering nodes are then two Poisson point processes with intensities λ_S and λ_P. This argument also holds if we consider a transmission from a Secondary node. The interference distribution at the origin is thus the same for both Primary and Secondary networks.

B. Model 2: the modified Matèrn

In this Section, we compute the throughput for the second model. First, we find the Frame Error Rate for the modified Matèrn point process. We consider FER for a transmission from a Primary. Let ξ an exponential r.v. with parameter 1, we have:

$$\begin{aligned} FER = \mathbb{P}(SIR < \theta) &= \mathbb{P}\left(\xi < \frac{\theta(I_P + I_{S \to P})}{P_P l(d)}\right) \\ &= 1 - \mathbb{E}\left[\exp\left\{-\frac{\theta}{P_P l(d)} I_{S \to P}\right\} \exp\left\{-\frac{\theta}{P_P l(d)} I_P\right\}\right] \qquad (15) \end{aligned}$$

It is not possible to compute this quantity analytically as the distribution of $I_{S \to P}$ is unknown. Moreover, I_P and $I_{S \to P}$ are dependent. As an approximation, we assume that $I_{S \to P}$ and I_P are independent. We will show through simulations that this assumption does not bias the results. Using this assumption, we get:

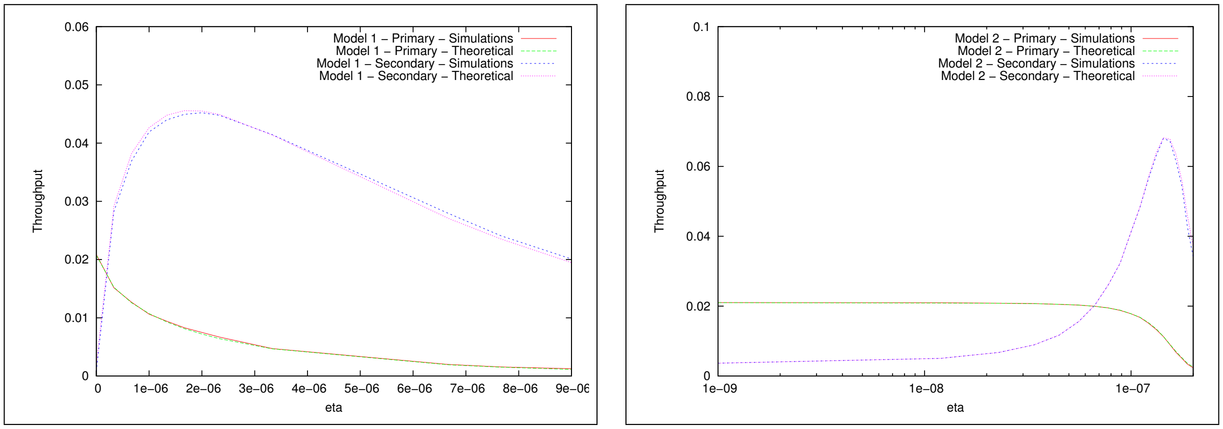
$$FER = 1 - \mathbb{E}\left[\exp\left\{-\frac{\theta}{P_P l(d)} I_{S \to P}\right\}\right] \mathbb{E}\left[\exp\left\{-\frac{\theta}{P_P l(d)} I_P\right\}\right]$$

where we have:

$$\mathbb{E}\left[\exp\left\{-\frac{\theta}{P_P l(d)} I_P\right\}\right] = \exp\left\{-\lambda_P 2\pi \int_0^{+\infty} \frac{\theta l(r)}{l(d) + \theta l(r)} r dr\right\} \qquad (16)$$

This quantity is computed in the proof of Proposition 2. We have shown that $I_{S \to P}$ can be approximated by a log-normal distribution, so we use the Laplace transform of the log-normal distribution to approximate $\mathbb{E}\left[\exp\left\{-\frac{\theta}{P_P l(d)} I_{S \to P}\right\}\right]$. In order to obtain the Frame Error Rate in the secondary network, it suffices to substitute $P_P l(d)$ by $P_S l(d)$ in the Formula (15). In Formula (18), the Laplace transform of I_P is taken in $\mathbb{R}^2 \backslash b(T, h_p)$ instead of $\mathbb{R}^2$ since we cannot have a Primary lying at a distance less than h_P from a Secondary transmitter.

Proposition 3. *Approximation of throughputs for Primary and Secondary networks are:*



(a) Model 1. $\epsilon = 5.0e-02$. η varies.

(b) Model 2. $\epsilon = 5.0e-02$. η varies.

Fig. 3. Throughput for model 1 and 2. $l(u) = min\left(\left(\frac{\beta}{4\pi u}\right)^{\alpha}, 1\right)$. $\beta = 0.346$ meters (wavelength). $\alpha = 3$. $\lambda_P = 0.00005$. $\lambda_S = 0.001$. $P_S = P_P = 40mW$. $\theta = 10$. The distance between receiver and transmitter is $d = 10$ meters. λ_S for model 1, h_S and h_P for model 2, are computed according to the method described in Section III-A and III-B. We considered $5,000$ samples.

$$throughput_{Primary} = \lambda_P \exp\left\{-\lambda_P 2\pi \int_0^{+\infty} \frac{\theta l(r)}{l(d) + \theta l(r)} r dr\right\} \times \mathbb{E}\left[\exp\left\{-\frac{\theta}{P_P l(d)} I_{S\to P}\right\}\right] \frac{1}{T} \quad (17)$$

$$throughput_{Secondary} = \lambda_S^{'} \exp\left\{-\lambda_P 2\pi \int_{\mathbb{R}^2 \setminus b(T,h_P)} \frac{\theta P_P l(\|x\|)}{P_S l(d) + \theta P_P l(\|x\|)} dx\right\} \times \mathbb{E}\left[\exp\left\{-\frac{\theta}{P_S l(d)} I_{S\to S}\right\}\right] \frac{1}{T} \quad (18)$$

where $\lambda_S^{'}$ is the intensity of the Matèrn point process given by equation (3) and where $I_{S\to P}$ is supposed to follow a log-normal distribution with mean and variance given by equations (9) and (10). $I_{S\to S}$ follows the same log-normal distribution as $I_{S\to P}$, but with diffrent parameters: in equations (9) and (10), $b(T, h_P)$ must be replaced by $b(T, h_S)$ in the first integral.

V. Throughput under interference constraint

For a given value of ϵ in Equation (4), we use the bound and approximation developed in Section III to determine parameters of the secondary network in such a way that transmissions from Secondary nodes satisfy the condition on interference. In Figures 3(a) and 3(b), we vary η of Equation (4) and we observe the throughput under this constraint. We also performed simulations varying ϵ rather than η. It led to the same behavior. In these two figures, we can observe that throughputs of the secondaries form a peak. This peak is due to the following phenomena. When η increases, the intensity of the simultaneous secondary transmitters increases, since the interference constraint becomes looser. There are therefore more transmitters and more frames received. When this intensity becomes high, interference generated by Secondaries becomes significant increasing the Frame Error Rate and decreasing the throughput. Values of the throughput may be used to tune parameters of the Secondaries. For model 1, Primary throughput suffers from secondary transmissions. Indeed, Primary throughput decreases whatever the value of η is. For model 2, Primary throughput is not impacted by secondary' transmissions until η reaches a threshold (approximately $\eta = 4.0e^{-8}$). The sensing mechanism guarantees that secondary transmitters are not too close to Primaries. For greater values of η, interference from Secondary' transmissions penalizes throughput in the Primary network. For this model, η (and consequently γ, h_S and h_P) should be chosen close to this threshold. It offers a good throughput for Secondaries without penalizing throughput of the Primaries.

VI. Conclusion

Obtaining the throughput for primary and secondary nodes in a cognitive radio network is of considerable interest. However, in order to obtain the throughput, it would be necessary to meet the interference constraint from secondary nodes to primary nodes. In addition, the interference level amongst primary nodes influences the throughput. Therefore, it becomes necessary to model the interference level for both primary and secondary nodes and obtain the region of operation with respect to acceptable interference. For this purpose, we first obtain bounds for the CCDF of interference level from secondary nodes to a primary node for Poisson point processes representing both primary and secondary nodes. In addition we model the CCDF of Secondary interference when secondary nodes are distributed according to a modified version of Matèrn point process which models the secondary transmitters by taking into account the medium access regulations. Subsequently, by finding the operation region of the secondary network that satisfies the interference constraint, analytical results are found for the throughput limits in the primary and secondary networks.

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Appendix

Proof: Proof of Proposition 1
The lower bound is computed as follows:

$$\mathbb{P}(I_{S\to P}\le\eta) = \mathbb{P}\left(P_S\zeta_1 l(\|X_1\|)\le \eta-\sum_{i=2}^{+\infty}P_S\zeta_i l(\|X_i\|)\right) \quad (19)$$

$$= \mathbb{P}\left(\zeta_1\le\frac{\eta-\sum_{i=2}^{+\infty}P_S\zeta_i l(\|X_i\|)}{P_S l(\|X_1\|)}\right) \quad (20)$$

$$= \mathbb{E}\Bigg[\left(1-\exp\left\{-\frac{\eta-\sum_{i=2}^{+\infty}P_S\zeta_i l(\|X_i\|)}{P_S l(\|X_1\|)}\right\}\right) \times \mathbb{1}_{\eta-\sum_{i=2}^{+\infty}P_S\zeta_i l(\|X_i\|)>0}\Bigg] \quad (21)$$

We set,

$$I^k_{S\to P} = \sum_{i=k}^{+\infty}P_S\zeta_i l(\|X_i\|) \quad (22)$$

$$\begin{aligned}&\mathbb{P}(I_{S\to P}\le\eta)\\ &=\mathbb{P}\left(I^2_{S\to P}\le\eta\right)-\mathbb{E}\left[\exp\left\{-\frac{\eta-I^2_{S\to P}}{P_S l(\|X_1\|)}\right\}\mathbb{1}_{I^2_{S\to P}<\eta}\right]\\ &=\mathbb{P}\left(\zeta_2\le\frac{\eta-I^3_{S\to P}}{P_S l(\|X_2\|)}\right)-\mathbb{E}\left[\exp\left\{-\frac{\eta-I^2_{S\to P}}{P_S l(\|X_1\|)}\right\}\mathbb{1}_{I^2_{S\to P}<\eta}\right]\\ &=\mathbb{P}\left(I^3_{S\to P}\le\eta\right)-\sum_{k=1}^{2}\mathbb{E}\left[\exp\left\{-\frac{\eta-I^{k+1}_{S\to P}}{P_S l(\|X_k\|)}\right\}\mathbb{1}_{I^{k+1}_{S\to P}<\eta}\right] \quad (23)\end{aligned}$$

By recurrence, we obtain for $n>1$:

$$\begin{aligned}&\mathbb{P}(I_{S\to P}\le\eta)\\ &=\mathbb{P}(I^n_{S\to P}\le\eta)-\sum_{k=1}^{n-1}\mathbb{E}\left[\exp\left\{-\frac{\eta-I^{k+1}_{S\to P}}{P_S l(\|X_k\|)}\right\}\mathbb{1}_{I^{k+1}_{S\to P}<\eta}\right] \quad (24)\end{aligned}$$

and when $n\to+\infty$,

$$\begin{aligned}&\mathbb{P}(I_{S\to P}\le\eta)\\ &=1-\sum_{k=1}^{+\infty}\mathbb{E}\left[\exp\left\{-\frac{\eta-I^{k+1}_{S\to P}}{P_S l(\|X_k\|)}\right\}\mathbb{1}_{I^{k+1}_{S\to P}<\eta}\right] \quad (25)\end{aligned}$$

We apply the Campbell formula [6]:

$$\begin{aligned}&\mathbb{P}(I_{S\to P}\le\eta)\\ &=1-\lambda_S\int_{\mathbb{R}^2}\mathbb{E}^0\left[\exp\left\{-\frac{\eta-I^x_{S\to P}}{P_S l(\|x\|)}\right\}\mathbb{1}_{I^x_{S\to P}<\eta}\right]dx \quad (26)\end{aligned}$$

where $\mathbb{E}^0[.]$ is the expectation under Palm measure [6], [**?**], and

$$I^x_{S\to P} = \sum_{X_i\in\mathbb{R}^2\setminus\overline{b(-x,\|x\|)}}^{+\infty}P_S\zeta_i l(\|X_i\|) \quad (27)$$

where $b(-x,\|x\|)$ is the ball centered at $-x$ with radius $\|x\|$ and $\overline{A}$ is the closed set of A.
As the Poisson point process is stationary, we can use the following definition instead:

$$I^x_{S\to P} = \sum_{X_i\in\mathbb{R}^2\setminus\overline{b(0,\|x\|)}}^{+\infty}P_S\zeta_i l(\|X_i\|) \quad (28)$$

Moreover, from the Slivnyak's theorem [6], we have:

$$\begin{aligned}&\mathbb{E}^0\left[\exp\left\{-\frac{\eta-I^x_{S\to P}}{P_S l(\|x\|)}\right\}\mathbb{1}_{I^x_{S\to P}<\eta}\right]\\ &=\exp\left\{-\frac{\eta}{P_S l(\|x\|)}\right\}\mathbb{E}\left[\exp\left\{\frac{I^x_{S\to P}}{P_S l(\|x\|)}\right\}\mathbb{1}_{I^x_{S\to P}<\eta}\right]\end{aligned}$$

The bound turns out as follows:

$$\begin{aligned}&\mathbb{E}\left[\exp\left\{\frac{I^x_{S\to P}}{P_S l(\|x\|)}\right\}\mathbb{1}_{I^x_{S\to P}<\eta}\right]\\ &=\mathbb{E}\left[\prod_{i=1}^{+\infty}\left(\exp\left\{\frac{\zeta_i P_S l(\|X_i\|)}{P_S l(\|x\|)}\right\}\mathbb{1}_{\|X_i\|>\|x\|}+\mathbb{1}_{\|X_i\|\le\|x\|}\right)\mathbb{1}_{I^x_{S\to P}<\eta}\right]\\ &\le\mathbb{E}\left[\prod_{i=1}^{+\infty}\left(\exp\left\{\frac{\zeta_i l(\|X_i\|)}{l(\|x\|)}\right\}\mathbb{1}_{\|X_i\|>\|x\|}\mathbb{1}_{P_S\zeta_i l(\|X_i\|)<\eta}+\mathbb{1}_{\|X_i\|\le\|x\|}\right)\right]\end{aligned}$$

We use the p.g.f.l. of the Poisson point process defined as:

$$\mathbb{E}\left[\prod_{i=1}^{n}v_x(X_i)\right]=\exp\left\{-\lambda_S\int_{\mathbb{R}^2}(1-v_x(u))\,du\right\}$$

with

$$v_x(u)=\left(\exp\left\{\frac{\zeta l(\|u\|)}{l(\|x\|)}\right\}\mathbb{1}_{\|u\|>\|x\|}\mathbb{1}_{P_S\zeta l(\|u\|)<\eta}+\mathbb{1}_{\|u\|\le\|x\|}\right)$$

and we obtain:

$$\begin{aligned}&\mathbb{E}\left[\exp\left\{\frac{I^x_{S\to P}}{P_S l(\|x\|)}\right\}\mathbb{1}_{I^x_{S\to P}<\eta}\right]\\ &\le\exp\Bigg\{-\lambda_S\int_{\mathbb{R}^2}(1\\ &-\mathbb{E}\left[\left(\exp\left\{\frac{\zeta l(\|u\|)}{l(\|x\|)}\right\}\mathbb{1}_{\|u\|>\|x\|}\mathbb{1}_{P_S\zeta l(\|u\|)<\eta}+\mathbb{1}_{\|u\|\le\|x\|}\right)\right])\,du\Bigg\}\\ &=\exp\Bigg\{-\lambda_S\int_{\mathbb{R}^2}(1-\mathbb{1}_{\|u\|\le\|x\|})\\ &-\mathbb{E}\left[\exp\left\{\frac{\zeta l(\|u\|)}{l(\|x\|)}\right\}\mathbb{1}_{P_S\zeta l(\|u\|)<\eta}\right]\mathbb{1}_{\|u\|>\|x\|}du\Bigg\}\\ &=\exp\Bigg\{-\lambda_S\int_{\|u\|>\|x\|}(1-\mathbb{E}\left[\exp\left\{\frac{\zeta l(\|u\|)}{l(\|x\|)}\right\}\mathbb{1}_{P_S\zeta l(\|u\|)<\eta}\right])\,du\Bigg\}\end{aligned}$$

We obtain,

$$\mathbb{E}\left[\exp\left\{\frac{\zeta l\left(\|u\|\right)}{l\left(\|x\|\right)}\right\}\mathbb{1}_{P_S\zeta l(\|u\|)<\eta}\right]=\frac{1-\exp\left\{-\frac{\eta}{P_S}\left(\frac{1}{l(\|u\|)}-\frac{1}{l(\|x\|)}\right)\right\}}{1-\frac{l(\|u\|)}{l(\|x\|)}}$$

Putting all of these together and changing for polar coordinates, we obtain:

$$\mathbb{P}\left(I_{S\to P}\leq\eta\right)\geq 1-2\pi\lambda_S\int_0^{+\infty}\exp\left\{-\frac{\eta}{P_S l(r)}\right\}$$
$$\times\exp\bigg\{-\lambda_S 2\pi\int_r^{+\infty}(1-\frac{1-\exp\left\{-\frac{\eta}{P_S}\left(\frac{1}{l(w)}-\frac{1}{l(r)}\right)\right\}}{1-\frac{l(w)}{l(r)}})wdw\bigg\}rdr \tag{29}$$

The upper bound on the complementary cumulative distribution function is then:

$$\mathbb{P}\left(I_{S\to P}\geq\eta\right)\leq 2\pi\lambda_S\int_0^{+\infty}\exp\left\{-\frac{\eta}{P_S l(r)}\right\}$$
$$\times\exp\bigg\{-\lambda_S 2\pi\int_r^{+\infty}\bigg(1-\frac{1-\exp\left\{-\frac{\eta}{P_S}\left(\frac{1}{l(w)}-\frac{1}{l(r)}\right)\right\}}{1-\frac{l(w)}{l(r)}}\bigg)wdw\bigg\}rdr$$

■

Proof: Proof of Proposition 2

First, we compute the Frame Error Rate for a node at the origin and receiving a frame from a primary node at distance d. We use the definition and method developped in [7]:

$$FER = \mathbb{P}\left(SIR<\theta\right) \tag{30}$$

The SIR is the ratio of the power received from the transmitter and the sum of the interference generated by the Primary and Secondary nodes. For a transmission from a Primary, we get:

$$FER=\mathbb{P}\left(\frac{\xi P_P l(d)}{I_{S\to P}+I_P}<\theta\right)$$
$$=\mathbb{P}\left(\xi<\frac{\theta}{P_P l(d)}\left(I_{S\to P}+I_P\right)\right) \tag{31}$$
$$=1-\mathbb{E}\left[\exp\left\{-\frac{\theta}{P_P l(d)}\left(I_{S\to P}+I_P\right)\right\}\right] \tag{32}$$

As $I_{S\to P}$ and I_P are independent we get

$$FER=1-\mathbb{E}\left[\exp\left\{-\frac{\theta}{P_P l(d)}I_{S\to P}\right\}\right]\mathbb{E}\left[\exp\left\{-\frac{\theta}{P_P l(d)}I_P\right\}\right] \tag{33}$$

$$\mathbb{E}\left[\exp\left\{-\frac{\theta}{P_P l(d)}I_P\right\}\right]$$
$$=\mathbb{E}\left[\exp\left\{-\frac{\theta}{P_P l(d)}\sum_{i=1}^{+\infty}P_P\xi_i l\left(\|Y_i\|\right)\right\}\right] \tag{34}$$
$$=\mathbb{E}\left[\prod_{i=1}^{n}\exp\left\{-\frac{\theta}{P_P l(d)}P_P\xi_i l\left(\|Y_i\|\right)\right\}\right] \tag{35}$$

We use the p.g.f.l. of the Poisson point process, we get:

$$\mathbb{E}\left[\exp\left\{-\frac{\theta}{P_P l(d)}I_P\right\}\right]$$
$$=\exp\left\{-\lambda_P\int_{\mathbb{R}^2}\left(1-\mathbb{E}\left[\exp\left\{\frac{-\theta}{P_P l(d)}P_P\xi l\left(\|y\|\right)\right\}\right]\right)dy\right\}$$
$$=\exp\left\{-\lambda_P 2\pi\int_0^{+\infty}\left(1-\mathbb{E}\left[\exp\left\{\frac{-\theta}{P_P l(d)}P_P\xi l(r)\right\}\right]\right)rdr\right\}$$
$$=\exp\left\{-\lambda_P 2\pi\int_0^{+\infty}\frac{\theta l(r)}{l(d)+\theta l(r)}rdr\right\}$$

We use the same approach to compute $\mathbb{E}\left[\exp\left\{-\frac{\theta}{P_P l(d)}I_{S\to P}\right\}\right]$:

$$\mathbb{E}\left[\exp\left\{-\frac{\theta}{P_P l(d)}I_{S\to P}\right\}\right]=\exp\left\{-\lambda_S 2\pi\int_0^{+\infty}\frac{\theta P_S l(r)}{P_P l(d)+\theta P_S l(r)}rdr\right\}$$

Throughputs for Primary and Secondary networks are then:

$$throughput_{Primary}$$
$$=\lambda_P\exp\left\{-\lambda_S 2\pi\int_0^{+\infty}\frac{\theta P_S l(r)}{P_P l(d)+\theta P_S l(r)}rdr\right\}$$
$$\times\exp\left\{-\lambda_P 2\pi\int_0^{+\infty}\frac{\theta l(r)}{l(d)+\theta l(r)}rdr\right\}\frac{1}{T} \tag{36}$$

$$throughput_{Secondary}$$
$$=\lambda_S\exp\left\{-\lambda_S 2\pi\int_0^{+\infty}\frac{\theta l(r)}{l(d)+\theta l(r)}rdr\right\}$$
$$\times\exp\left\{-\lambda_P 2\pi\int_0^{+\infty}\frac{\theta P_P l(r)}{P_S l(d)+\theta P_P l(r)}rdr\right\}\frac{1}{T} \tag{37}$$

■