Analysis of Inter-Vehicle Communication to Reduce Road Crashes

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***Abstract*—The aim of this paper is to study the contribution of IVC to reduce the number of secondary collisions caused by an accident. We assess the impact of broadcast protocols used to disseminate warning messages and the impact of IVC penetration ratio on the number of collisions. There are two main contributions in this paper. Firstly, it addresses the analytical evaluation of broadcast protocols in VANETs. We propose a general framework based on point processes to evaluate the performance of this protocol. Our approach is based on the theory of point processes and Palm Calculus. We derive, for a generic *Frame Error Rate* function, simple formulas for computing the delay required to propagate messages. These formulas hold for any *FER* function, enabling the comparison of broadcast performance under different radio propagation environments. In the second part of the paper, we propose an analytical evaluation of the number of vehicles involved in a collision. This analysis allows us to study the impact of radio technology penetration ratio and the impact of safety messages delivery delays on the number of collisions.**

***Index Terms*—Vehicular Ad Hoc Network, Performance Evaluation, Broadcast Protocols, Road Safety.**

I. Introduction

In recent years, Inter-Vehicle Communication (IVC) has become an intense research area, as part of Intelligent Transportation Systems. It assumes that all, or, a subset of the vehicles is equipped with radio devices, enabling communication between them. These communications usually use ad hoc modes. This allows a vehicle to communicate directly with another vehicle without the use of any dedicated infrastructure (Base Station, Access Point, etc.). Although classical 802.11 can be used for IVC, specific technologies such as IEEE 802.11p [13](also referred to as Wireless Access in Vehicular Environments, WAVE) show a great deal of promise. This standard includes data exchanges between vehicles and between infrastructure and vehicles with a greater radio range than classical 802.11. Also, by using ad hoc mode of these radio technologies (all the 802.11 technologies have an ad hoc mode), we gain the advantage that the scope of communications is not just limited to the radio range. Hence, vehicles can act as routers, i.e. implement forwarding and routing algorithms, and form a Multi-Hop wireless Ad-hoc NETwork (MANET), also called Vehicular Ad hoc NETwork (VANET). Such functionality provides good dissemination of messages. This paper focuses on the dissemination of warning and control information [21], [1]. This allows a vehicle to obtain and disseminate information about accidents, congestions, and road surface conditions coming from other vehicles. Such applications rely on broadcast algorithms [7], [11], [18], [33]. These algorithms are given the task of disseminating warning messages quickly and efficiently through the network. Thus, the performances of these algorithms are crucial.

The aim of this paper is to study the contribution of IVC in the reduction of secondary collisions following an initial crash. We assess the impacts of early warning communication on the number of collisions. There are two main contributions of this paper.

Firstly, we address the analytical evaluation of the time required to disseminate warning messages in a string of vehicles. We focus on the most popular broadcast algorithm in VANETs and propose a general analytical framework to evaluate it. Our approach is based on the theory of point processes and Palm Calculus [31], [5]. We derive simple formulas which hold for any *Frame Error Rate* (*FER*) function, enabling the comparison of broadcast performances under different radio propagation environments. Previous works deal with the classical Boolean model (where vehicles have a fixed radio range) or use a discrete space to model vehicle locations [4], [10], [35]. We improve these models as we consider a generic radio model.

Secondly, we propose an analytical evaluation of the number of vehicles involved in a crash. This evaluation relies on the impact of radio technology penetration ratio and on the delivery delay of warning messages. To the best of our knowledge, it is the first time that such a study is proposed: previous works do not deal with the delivery delay of the messages [32], [29], [26], [28].

The remainder of this paper is organized as follows. Section II provide an overview of broadcast protocols in VANETs, related works on performance evaluation and the contribution of IVC to road safety. Section III presents the model and numerical evaluation of the broadcast protocol. The analytical study and numerical evaluation of the mean number of expected collisions are presented in Section IV. Finally, concluding remarks and comments are given in Section V.

II. Related works

A. Presentation of broadcast protocols

The simplest broadcast mechanism consists for a node in broadcasting the message right after the first reception. This mechanism has the benefit of being simple, but it creates the famous storm problem [34], also named broadcast flooding, as it generates a great number of retransmissions and receptions. The goal of an efficient broadcast protocol is thus to minimize the number of transmissions while keeping a high probability of reception.

In [34], broadcast protocols are categorized with respect to criteria used by a potential forwarder to cancel its own retransmission: distance to the emitter, number of receptions (a node receiving n times the same message cancels its retransmission), a probability (a node cancels its retransmission with a given probability), and node locations. The latter assumes that nodes are able to know their geographical locations. A node forwards the broadcast message when the additional coverage is greater than a predefined threshold. This approach is shown to be the most efficient, as it eliminates rebroadcast duplications without compromising reachability.

This algorithm has been improved and adapted in the context of VANETs. Most of the broadcast protocols favor the farthest nodes from the current emitter as the next forwarder. It maximizes the coverage area and minimizes the number of redundant receptions. For instance, in [3], [10], a vehicle retransmits the message according to a certain probability. This probability is increasing with distance from the emitter and thus farther nodes are likely to be selected as forwarder.

In [2], [8], [17], the farthest receiver is systematically the next forwarder, but the way it is selected differs from one protocol to another. In [8], each node is supposed to know its neighborhood (IDs and locations of the vehicles within its radio range). A forwarder selects in its neighborhood the farthest node in the broadcast direction. A field in the message indicates the ID of the node responsible for the next retransmission. In [2] and [17], upon receiving a frame, a node triggers a retransmission timer (a blackburst in [17]) with a duration decreasing with the distance from the emitter. In consequence, the farthest node retransmits first. Upon receiving this broadcast, the other nodes cancel their own transmission.

In this paper, we consider a comparable approach. We assume that vehicles know their geographical locations. We assume data fusion provides sufficiently accurate relative position of each vehicle [15], [27] for the broadcast protocol to work properly. The algorithm is as follows. We study the propagation of the message in a given direction with regard to the road (upstream or downstream). In the following, x, y and z represent locations of three vehicles on a road or highway. The selection of the vehicles/nodes which forward the message depends on the vehicles' location. When a vehicle at y receives for the first time a broadcast message from a vehicle at x with $x < y$, it triggers a timer. The initial value of the timer, denoted $timer(|y-x|)$, is decreasing with distance ($|y - x|$). If the vehicle at y did not receive the same message from an other vehicle at z with $z > y$ at the timer expiration, it retransmits the message, otherwise it cancels its transmission.

B. Performance evaluation of brodacast protocols in VANETs

Some analytical works exists on VANET, describing their structural properties such as connectivity, route lifetime and capacity [22], [36], [19], [23], but there are only a few analytical studies about performance evaluation of broadcasts protocols in VANETs. In [35], the effects of broadcast flooding and several schemes to reduce redundant broadcasts in Ad-Hoc networks are analyzed. [4] proposes a model to assess the overhead, coverage and latency characteristics of a particular broadcast algorithm for VANETs. The model used simplistic radio assumptions, where the radio range of the nodes is fixed and identical for all nodes. Moreover, the considered algorithm is very simple and not realistic in the context of VANETs. In [10], the authors develop an analytical framework to study broadcast performance, and derive several metrics relevant to the dissemination of safety messages. There are two limitations with their model: a discrete space is used to represent vehicle locations, and they assume an ideal radio environment with a fixed radio range.

C. Enhance road safety with early warning communication

Warning messages are essentials for road safety because they allow vehicles to react to indirectly detectable events. Although some works have been concerned with the study of collision on the road [9], [16], little research efforts have been devoted to study the benefit of warning communications in collision reduction. [32] examines the reduction of the Average Accident Interval (AAI) by means of communication. It concludes that a high penetration ratio of communication system ($> 60\%$) is necessary in order to increase significantly the AAI. Other works concerning the joint study of communications and sensors lead to similar results [29]. [26] shows that warning communications allow an enhancement of the safety-capacity relation thus indicating a reduction in the number of collisions for a constant road capacity. [28] evaluates the impact of communication when considering the use of various sensors in a classical chain collision scenario. It indicates that with a 50% penetration, almost all heavy collisions are avoided. A similar scenario has been considered in this paper but we assume an initial collision that activates a minimal warning communication system (the system sends only the warning message, no add-on localization or sensing messages are sent).

III. Broadcast performance evaluation

A. Model

We use a homogeneous Poisson point process with parameter λ (λ is the mean number of vehicles per kilometer) to model vehicle positions. Indeed, it has been shown that for certain densities of vehicles, vehicle positions follow a Poisson point process [25], [24], [14], [30]. It corresponds to *free flow* conditions, where drivers can choose their own speed. Basically, a point process consists in a sequence of points randomly distributed on the line, each point representing a vehicle position. A Poisson point process N with intensity λ is defined by two properties (a more detailed presentation of the Poisson process can be found in [31], [12]):

- The number of points in two disjoint intervals of $I\!R$ form two independent random variables.
- The number of points in an interval $[a, b] \subset I\!R$ with $b > a$, denoted $N([a, b])$ follows a discrete Poisson distribution with parameter $\lambda(b-a)$:

$$\mathbb{P}\left(N([a,b]) = k\right) = \frac{(\lambda(b-a))^k}{k!} e^{-\lambda(b-a)} \quad (1)$$

The point process is distributed on a line rather than in a plane as the vehicle radio ranges are significantly larger than the road width. The point process can model one or several lanes, and one or two directions. The considered Poisson point process is then the result of the superposition of several independent Poisson processes, one for each lane/direction. This model also applies to cases where all, or l ($0 < l \leq 1$) a proportion of vehicles are equipped with radio devices. As the considered process modeling vehicles is Poisson, the thinning of the process representing the equipped vehicles is also Poisson.

We assume that a transmission from one vehicle is properly received by a vehicle at distance x with probability $1 - p(x)$. The function $p(.)$ is the *Frame Error Rate* (*FER*) with respect to distance. This function takes its value in $[0, 1]$ with $p(0) = 0$. It is supposed to be continuous and $\int_0^{+\infty}(1 - p(x))dx < +\infty$. This last assumption involves that $p(x)$ tends to 1 as $x \to +\infty$ and guarantees that all the probabilistic quantities (esperance) are finite. We use the same function $p(.)$ for transmissions from all vehicles. Receptions are supposed to be independent between vehicles.

The point process used to model all vehicle positions is built in two steps. Firstly, in Section III-B, we consider a Poisson point process with parameter λ. It is used to derive the *probability density function* (*pdf*) of the distance between the successive forwarders of the broadcasted message. Secondly, in Section III-C, we consider a point process where the distance between the successive points are distributed according to a normalizing version of this *pdf*. It models only the vehicles which retransmit the warning message. Therefore, a second point process modeling all the other vehicles is superposed to this point process.

B. First step.

We will first look at vehicles involved in the progression of broadcasted messages. According to the broadcast protocol, we can distinguish two kinds of emitters. This distinction is easier to understand with the example depicted in Figure 1. Node 0 initiates the message. This first broadcast is received by nodes 1, 2, 3 and 4. Node 4, being the farthest from 0 is the first node to retransmit the message. The message is received by nodes 5, 6 and 8, the latter rebroadcasts the message. Assuming that node 3 does not receive the retransmission from 4, it will also retransmit the message (not shown in the figure). We can thus distinguish a first set of emitters contributing to the fast propagation of the message (nodes 4 and 8 in our example), and the emitters which retransmit because they did not receive the message from nodes ahead (node 3 in our example). The first set of emitters

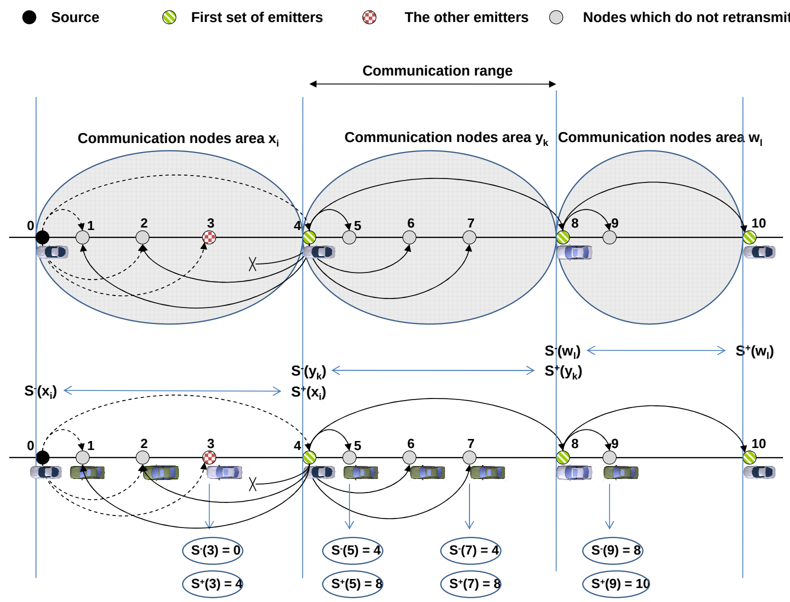


Fig. 1. Example of a broadcast scenario.

is denoted $(T_i)_{i\geq 0}$ in this paragraph. Formally, a sample of the random variable T_i corresponds to the location of one of the fowarders. But, in order to limit the number of notations, we also use T_i to represent the point (the forwader): "point T_i" means "the point at location T_i".

Let Φ be a homogeneous Poisson point process with intensity λ. We build a thinning $(T_i)_{i\geq 0}$ of the Poisson point process, i.e. we select a subset of the points in Φ, as follows. Let $T_0 = 0$ be the location of the node which initiates the broadcast. We denote by T_1, the farthest node which received the message from T_0. As the farthest receiver is the next forwarder, T_1 will retransmit the message. The farthest node which received the message from T_1 is denoted T_2, and so on. The sequence $(T_i)_{i\geq 0}$ is thus defined recursively. T_i is then formally defined as the farthest node which received the message from T_{i-1}. The sequence $(T_i)_{i\geq 0}$ corresponds to the fastest progression of the message. In an ideal world where vehicles have a perfect radio range R ($p(x) = 0$ for $x < R$ and $p(x) = 1$ otherwise), the nodes $(T_i)_{i\geq 0}$ are the only emitters. Indeed, a node of Φ not belonging to $(T_i)_{i\geq 0}$ will receive the message from a node T_i (in the source direction), and the retransmission from T_{i+1} will cancel its own transmission.

After the initial transmission from T_0, nodes which have received the messages from T_0 forms an inhomogeneous point process. Indeed, thanks to the properties of the Poisson point processes, an independent thinning of a Poisson process is still Poisson (the thinning is independent if each point is selected independently of the others). In our case, it leads to an inhomogeneous Poisson process as the probability to select a point depends on its distance to 0. An inhomogeneous Poisson process is characterized by its intensity measure, i.e. a measure representing the mean number of points as a function of a subset A, $A \subset I\!R^+$ (more formally, $A \in \mathcal{B}(I\!R^+)$, where $\mathcal{B}(I\!R^+)$ is the Borel algebra in $I\!R^+$). In our case, the intensity measure is given by:

$$\Lambda(A) = \lambda \int_A (1 - p(x))\, dx \qquad (2)$$

The distribution of the number of points in $A \subset I\!R^+$ for this point process (vehicles receiving the message from T_0), follows a discrete Poisson distribution with parameter $\Lambda(A)$. It is then easy to deduce the *pdf* of the distance between the initial transmitter at $T_0 = 0$ and the farthest receiver at T_1. The distance T_1 will be less than x if and only if there is no receiver at distance greater than x, in other words if there is no point of the inhomogeneous process in $[x, +\infty)$. The *cumulative* *distribution function* (*cdf*) of T_1 is thus given by:

$$\mathbb{P}(T_1 \leq x) = e^{-\Lambda([x,+\infty))} \quad (3)$$

From the derivative of the *cdf* of T_1, we obtain its *pdf*:

$$f_{T_1}(x) = \lambda(1-p(x))e^{-\Lambda([x,+\infty))} + q\delta_0 \quad (4)$$

The *pdf* is composed of a continuous term and a singularity at 0 ($\delta_.$ is the Dirac measure) corresponding to the event that there is no receiver: $q = e^{-\Lambda([0,+\infty))}$.

C. Second step

In the computation of the mean delay, we will suppose that there is a point/vehicle at a given location (the initial transmitter or a typical vehicle at distance x). The presence of this point impacts the distribution of the point process. Intuitively, Palm calculus gives a formal mathematical framework and a set of tools to compute quantities relative to point processes conditionally to the presence of a point at a given location (generally at the origin). The probability measure being different under this assumption, expectation under Palm measure is then denoted $E^x_N[.]$ meaning (intuitively) that a point belonging to process N is located at x. Palm calculus applies only on stationary point processes. The point process $(T_i)_i$ was not stationary. Therefore, in this paragraph, we build a new stationary point process $(S_i)_i$ to represent locations of the forwarders.

We use the *pdf* of $T_1 - T_0$ to build a point process $(S_i)_{i\in\mathbb{N}}$ where the distance between the successive forwarders $S_{i+1} - S_i$ have the same distribution as $T_1 - T_0$. Nevertheless, we do not suppose that $S_{i+1} - S_i$ can be equal to 0, i.e. we consider that there is always a vehicle receiving the message. Therefore, we consider a normalized version of formula (4) for the *pdf* of $S_{i+1} - S_i$:

$$f_{S_{i+1}-S_i}(x) = \frac{1}{1-q}\lambda(1-p(x))e^{-\Lambda([x,+\infty))} \quad (5)$$

This process, denoted Φ_S, is thus stationary with intensity

$$\lambda_S = \frac{1}{E[S_{i+1} - S_i]} \quad (6)$$

A second stationary point process Φ_O models the other nodes. It is modeled by an independent Poisson point process of intensity $\lambda - \lambda_S$.

The global process describing all the nodes is thus a stationary point process $\Phi = \Phi_S \cup \Phi_O$ with intensity λ. λ_S is the mean number of vehicles forwarding the message (taking into account only forwarders selected as the "furthest receivers" after each transmission, therefore the actual number of forwarders can be greater), and $\lambda - \lambda_S$ is the intensity of the other vehicles. λ is thus the mean number of vehicles per kilometer (all vehicles). We also define for a point x of Φ_O, two points $S^-(x)$ and $S^+(x)$ of Φ_S downstream and upstream x. More formally, $S^-(x)$ (resp. $S^+(x)$) is the closest point of Φ_S from x with $S^-(x) < x$ (resp. $S^+(x) > x$). In Figure 1, we show our different notations for the previous example.

D. Delay

We estimate delivery delay of the message for a node located at $x \in \mathbb{R}^+$, and at distance x from the node which initiates the broadcast. We suppose that the message has been initially broadcasted by a node located at 0 at time $t = 0$. We propose a lower and an upper bound on this delay. Let d_S be the mean delay of a retransmission, i.e. the mean delay between the reception and the retransmission for a node of Φ_S. We obtain,

$$d_S = E^0_{\Phi_S}[timer(S_1)] + T \quad (7)$$

where $timer(.)$ is the duration of the timer with regard to the distance. T is a constant representing the time to physically send the message, i.e. times to access the medium, sent the frame, etc.

a) Lower bound on the delay (d_{min}): In the best case, node x receives directly the message from $S^-(x)$. If a_x is the mean number of emitters in $\Phi_S \cap [0, x]$ and, d_S the mean delay added by a transmitter, the mean delay at x denoted $delay(x)$ is then $(a_x - 1)d_S$ with $a_x \geq 1$ as it counts systematically the first emitter at 0. We subtract 1 from a_x because the initial emitter does not add any delay. The computation of a_x is not trivial. It is formally defined as:

$$a_x = E^0_{\Phi_S}\left[\sum_{x_i\in\Phi_S} \mathbb{1}_{x_i\in[0,x]}\right] \quad (8)$$

where $\mathbb{1}_{Condition}$ is the indicator function equals to 1 if $Condition$ is true and 0 otherwise.

We add to this bound a term taking into account the fact that the node at x does not receive the message from $S^-(x)$ due to a frame error but from $S^+(x)$. It adds d_S to the delay on average. It is still a lower bound since we do not take into account the fact that transmission may be received from an emitter of Φ_O with a greater delay. Formally, the probability for x to receive the message from $S^-(x)$ is given by:

$$E^0_{\Phi_S}\left[\left(1 - p\left(x - S^-(x)\right)\right)\right] \quad (9)$$

It may be estimated by

$$E^0_{\Phi_O}\left[\left(1-p\left(S^-(0)\right)\right)\right] \tag{10}$$

Formula (10) gives the probability for a typical node of Φ_O to receive the message from the previous forwarder $S^-(0)$. It is a classical change in Palm calculus. We shift the processes Φ_S and Φ_O in such a way that the point at x is shifted at the origin O. It does not change the distribution of the distance between this point and the forwarders ($S^-(0)$), and allow us to obtain a quantity which does not depend on the distance x anymore. We get,

$$\begin{aligned} delay(x) &\geq d_S(a_x-1)E^0_{\Phi_O}\left[1-p(S^-(0))\right] \\ &\quad +d_S a_x E^0_{\Phi_O}\left[p(S^-(0))\right] \\ &= d_S\left(a_x-1+E^0_{\Phi_O}\left[p(S^-(0))\right]\right) \\ &\geq d_S\Big(max(\lambda_S x-1,0) \\ &\quad +E^0_{\Phi_O}\left[p(S^-(0))\right]\Big) \\ &= d_{min} \end{aligned} \tag{11}$$

In the last inequality, we used $max(\lambda_S x-1,0)$ as a lower bound of a_x-1. From the Neveu's exchange formula of two Palm measures (see for instance [5], formula (1.3.4)), we get:

$$\begin{aligned} E^0_{\Phi_O}\left[p(S^-(0))\right] &= \lambda_S E^0_{\Phi_S}\left[\int_0^{S_1} p(u)du\right] \\ &= \lambda_S \int_0^{+\infty}\int_0^v p(u)du f_{S_1}(v)dv \end{aligned} \tag{12}$$

Under Palm expectation $S_0=0$, which is why we use $f_{S_1}(v)$ rather than $f_{S_1-S_0}(v)$.

b) Upper bound on the delay (d_{max}): The lower bound supposed that x receives the message from $S^+(x)$ or $S^-(x)$. But, it may instead receive it from another node, different from $S^+(x)$ and $S^-(x)$. In the worst case, the delay generated by the last transmitter is $d_0 = timer(0)+T$ since $timer(.)$ is a decreasing function. There are thus 3 possibilities, for the first reception of node x:

- x receives the frame from $S^-(x)$ (with probability $E^0_{\Phi_O}[1-p(S^-(0))]$), the delay is then $(a_x-1)d_S$;
- x receives the frame from $S^+(x)$ (with probability $E^0_{\Phi_O}[p(S^-(0))(1-p(S^+(0)))]$), the delay is then $a_x d_S$;
- x receives the frame from an emitter in $]S^-(x),S^+(x)[$ (with a probability bounded by $E^0_{\Phi_O}[p(S^-(0))p(S^+(0))]$), an upper bound on the delay is then $(a_x-1)d_S+d_0$;

Here, we use $\lambda_S x$ as an upper bound on a_x-1. The upper bound is thus,

$$\begin{aligned} &delay(x) \\ &\quad\leq d_S(a_x-1)E^0_{\Phi}\left[1-p(S^-(0))\right] \\ &\qquad +d_S a_x E^0_{\Phi}\left[p(S^-(0))(1-p(S^+(0)))\right] \\ &\qquad +((a_x-1)d_S+d_0)E^0_{\Phi}\left[p(S^-(0))p(S^+(0))\right] \\ &\quad= d_S\left(a_x-1+E^0_{\Phi}\left[p(S^-(0))\right]\right) \\ &\qquad +(d_0-d_S)E^0_{\Phi}\left[p(S^-(0))p(S^+(0))\right] \\ &\quad\leq d_S\left(\lambda_S x+E^0_{\Phi}\left[p(S^-(0))\right]\right) \\ &\qquad +(d_0-d_S)E^0_{\Phi}\left[p(S^-(0))p(S^+(0))\right] \\ &\quad= d_{max} \end{aligned} \tag{13}$$

with

$$\begin{aligned} &E^0_{\Phi}\left[p(S^-(0))p(S^+(0)\right] \\ &\quad= \lambda_S E^0_{\Phi_S}\left[\int_0^{S_1} p(u)p(S_1-u)du\right] \\ &\quad= \lambda_S \int_0^{+\infty}\int_0^v p(u)p(v-u)du f_{S_1}(v)dv \end{aligned} \tag{14}$$

E. Model Evaluation

Firstly, we present the set of parameters and functions used for the simulation and analytical formulas, then we compare the results of the analytical model with simulations.

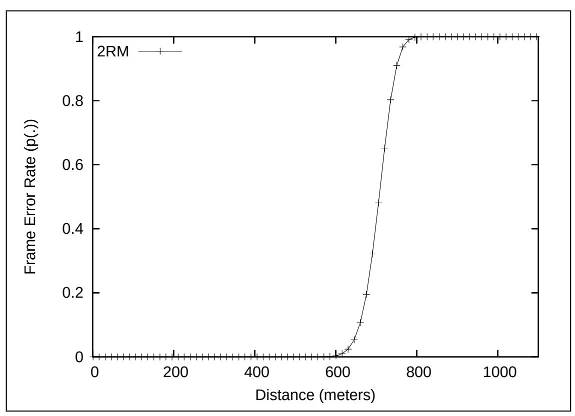


Fig. 2. *FER* functions.

1) Frame Error Rate FER *:* In order to set the *FER* function $p(.)$ according to the 802.11p standard, we use the measurement based model developed in [6]. The proposed model is based on the two-ray path loss model referred as 2RM. The model takes into account

Simulation Parameters	Numerical values	Simulation parameters	Numerical values
Frequency	5.9GHz	Number of samples (simulation)	$60,000$
Transmission Rate	3 Mb/s	Size of the observation window	from 0 to 25 km
Antenna heights	1.5 meters	Message length	100 bytes
Vehicle velocity	$v = 36.1$ m/s (130 km/h)	Vehicle length	$l = 5$ meters
Capacity	$c \in [1800, 3200]$ Veh/h	Mean intervehicle distance	$d_{inter} = \frac{v}{c/3600} - l$
Vehicle braking capacity	$\gamma = 0.8g$	Reaction time	$\tau = 1$ second
Distance covered during τ	$d_\tau = v\tau = 36.1$ m	Deceleration distance	$d_{dec} = \frac{v^2}{2\gamma} = 83.08$ m

TABLE I
PARAMETERS FOR THE SIMULATION AND MODEL EVALUATION.

wavelength of the 802.11p standard, heights, distances and gains of the two antennas (emitter and receiver), frame length, etc. Using the default parameters of the 802.11p standard listed in Table I, we obtain the *FER* plot in Figure 2. The radio range obtained with this model is consistent with respect to the expected radio range of 802.11p in a rural environment (up to 1 km).

2) Retransmission timer: The function $timer(.)$ must decrease with distance. We choose a function decreasing linearly with distance and where the timer is at most $1000\mu s$:

$$timer(x) = (-ax + b)1000 \tag{15}$$

With the chosen parameters, the maximum distance between the emitter and the receiver is 1100 meters, we get $a = -\frac{1}{1100}$ and $b = 1$. We add to this delay, the time T required by a forwarder to access the channel and transmits its frame. The MAC layer in 802.11p is similar to the IEEE 802.11e Quality of Service extension. Application messages are categorized into different ACs, where AC0 has the lowest and AC3 the highest priority. We consider here, that safety message use the highest priority $AC3$. For the highest priority, a frame must wait $AIFS = 2t_s$ (Arbitration Inter-Frame Space) where t_s is the slot time ($t_s = 16\ \mu s$). Next, the transmitter waits for a contention period randomly selected in the Contention Window (CW), where $CW = [0, 3 \times t_s]$ for the highest priority. So, it will be equal to $\frac{3}{2}t_s$ on average. Here, we suppose that a forwarder systematically wins access to the channel, as they use the highest priority and the $timer(.)$ function.The value of T is then $T = \frac{7}{2}t_s + 267 = 323\ \mu s$, where 267 μs is the time to transmit a frame of 100 bytes at 3 Mbit/s. Therefore, the mean delay d_S of a forwarder in $(S_i)_i$, is

$$\begin{aligned} d_S &= E[-a(S_{i+1} - S_i) + b]1000 + T \\ &= \left(\frac{-a}{\lambda_S} + b\right) 1000 + T \end{aligned} \tag{16}$$

where λ_S is defined in Section III-C. The maximum delay for a forwarder is given by $d_0 = timer(0) + T = 1323\mu s$.

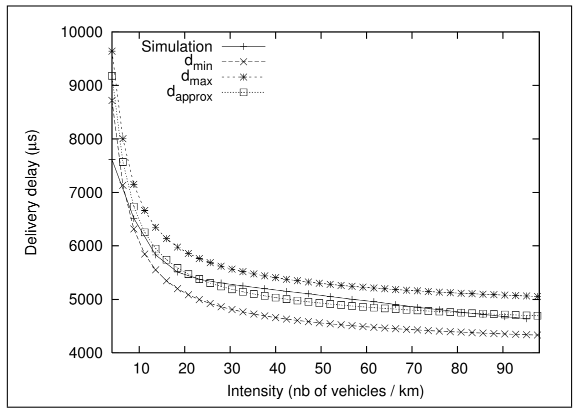


Fig. 3. Delivery delay for a node at 5 km from the first emitter.

3) Simulation and Results: We compare the theoretical bounds d_{min} and d_{max} given by equations (11) and (13) to simulations. We also consider an approximation of $delay(x)$ which consists in approximating $a_x - 1$ by $max(\lambda_S x - 0.5, 0)$ rather than $\lambda_S x$ in Formula (13):

$$\begin{aligned} & d_{approx} \\ &= d_S \left(max(\lambda_S x - 0.5, 0) + E^0_\Phi \left[p(S^-(0))\right]\right) \\ &\quad + (d_0 - d_S)\, E^0_\Phi \left[p(S^-(0))p(S^+(0))\right] \end{aligned} \tag{17}$$

We use a simulator coded in C. It uses the same assumptions and parameters as the analytical model. In Figure 3, we plotted the delivery delay for a vehicle 5 km away from the source of the message. The delay is perfectly bounded by the analytical formulas when the density is greater than 5 veh/km. It confirms the accuracy of our bounds. For a small density of vehicles (less than 5 veh/km), the difference is explained by the assumption of the stationarity of the point process Φ_S (the one modeling the successive forwarders) which does not hold.

IV. Number of collisions

A. Problem Statement

In order to analyze the usefulness of inter-vehicular warning communications, and to observe the impact of delivery delay on the number of collisions, we consider a string of vehicles whose leader has crashed with a stationary, heavy obstacle. The other drivers (human or computer) brake as soon as they are aware of the situation. They can be aware due to their own sensors or from a message broadcasted by the crashed vehicle. This section presents the notations and the capacity notion so as to study the impact of warning communications (after an initial collision) on the number of collisions.

1) Notations and assumptions: Let us consider a string of vehicles $(Veh_i)_{i \geq 0}$. To simplify the analysis, we assume the vehicle string to be homogeneous (i.e. all vehicles have the same characteristics). They are characterized by the following parameters:

- v, the velocity of the vehicles (in m/s),
- l, the length of the vehicles (in m),
- $d_{inter(i,i+1)}$, the interdistance between the i^{th} and the $(i+1)^{th}$ vehicle (in m),
- τ the reaction time of the drivers (in s),
- $d_\tau = \tau v$, the distance covered during the reaction time τ (in m),
- d_{dec} , the deceleration distance of the vehicles (in m),

Also, we suppose that when two vehicles collide, the length of the formed agglomerate is $2l$ (no compression).

2) Capacity: Just before a perturbation, the vehicle density (the number of vehicles / m) ρ is defined as:

$$\rho = \frac{1}{l + \bar{d}_{inter}} \tag{18}$$

where $\bar{d}_{inter}$ is the averaged initial interdistance. From this spatial repartition, we can define a temporal vehicle repartition as:

$$c = v\rho = \frac{v}{l + \bar{d}_{inter}} \tag{19}$$

where c is the capacity of the vehicles flow (number of vehicles/s) [26].

B. Strings of unequipped or fully equipped vehicles with ideal communication

In this section an analytical formulation of the number of collisions in a homogeneous string of unequipped vehicles is given. The interdistance between vehicles is supposed to be constant. This model allows us to introduce, by means of two simple examples, the computation of the number of collisions.

1) Vehicle String Without Warning Communications: Without warning communications, the driver of vehicle Veh_{i+1} brakes after seeing the brake lights of vehicle Veh_i. Reaction time, τ, is defined as the time required by the driver to perceive the danger and initiate braking. Therefore, Veh_1 starts braking after a reaction time τ.

As the vehicle string is homogeneous, when the front of the first's vehicle Veh_0 collides with the obstacle at time $t_{collision}$, the i^{th} vehicle is $(l + d_{inter}).i$ meters away from the obstacle. The braking of each vehicle is delayed by the reaction time of each preceding vehicle since each vehicle has a vision limited to its front vehicle. Thus, when Veh_i starts to break, $i\tau$ seconds have already passed after the initial collision occurred. The reaction time effect is a cumulative effect. The stopping distance of Veh_i is $d_{stop} = i.d_\tau + d_{dec}$ meters. When the first vehicles until vehicle $i-1$ have collided, the agglomerate is $i.l$ meters long. Therefore, Veh_i has to be at least $d_{stop} + i.l$ meters far from the obstacle at the time $t_{collision}$ to avoid collision with the $(i-1)^{th}$ vehicle:

$$x_i \geq i.d_\tau + d_{dec} + i.l \tag{20}$$

where x_i is i^{th} vehicle's front location (at time of initial collision). It can be rewritten as:

$$(l + d_{inter}).i \geq i.d_\tau + d_{dec} + i.l \tag{21}$$

The vehicle whose index is bigger or equal to i will not collide:

$$i \geq \frac{d_{dec}}{d_{inter} - d_\tau} \text{ for } d_{inter} \geq d_\tau \tag{22}$$

And the number of collided vehicles is:

$$C = \left\lfloor \frac{d_{dec}}{d_{inter} - d_\tau} \right\rfloor \tag{23}$$

where$\lfloor\ \rfloor$ means the integer part of the fraction. The number of collision is maximal (infinite if we consider an infinite string) when $d_{inter} = d_\tau$: the vehicles have not enough time to decelerate before colliding with the front vehicle.

2) Vehicle String With an Ideal Communication Technology: With an ideal communication technology (without delivery delay), all other vehicles are informed instantaneously as soon as the first vehicle has made an emergency stop. This scenario corresponds to the case where drivers can see at once the brake lights of all vehicles which are ahead of them. When a driver is alerted, he brakes after his reaction time τ. Compared to the previous case (without warning communication), here we notice that τ is not cumulative anymore. Thanks to communications, reaction times appear now as concurrent operation time. Equation (21) becomes:

$$(l + d_{inter}).i \geq d_\tau + d_{dec} + i.l \quad (24)$$

because the drivers break after d_τ (rather than $i.d_\tau$). Therefore, the number of collided vehicles is:

$$C = \left\lfloor \frac{d_{dec} + d_\tau}{d_{inter}} \right\rfloor \quad (25)$$

This number is finite as soon as we consider $d_{inter} \neq 0$.

C. String of partially equipped vehicles with a realistic communication technology and interdistance exponentially distributed

In this Section, we extend the previous model presented in [26], [20] in order to consider a string of vehicles only partially equipped with communication devices. Here, we also consider the delivery delay of warning messages and an inhomogeneous interdistance between vehicles. The distances between vehicles $d_{inter(i-1,i)}$ are now supposed to be exponentially distributed with parameter λ.

We assume that a vehicle has radio equipment with probability p, independently of the other vehicles. The leading vehicle (vehicle 0) is always equipped and emits instantaneously a warning message to all other equipped vehicles upon collision or emergency breaking. All vehicles which receive the warning from vehicle 0 brake after their reaction time τ. The other vehicles, not equipped with a radio, brake at a time τ after the vehicle in front of them brakes.

More formally, let us consider the i^{th} vehicle, and let X_i be its associated random variable. X_i describes the index of an equipped vehicle which is the nearest vehicle to the i^{th} vehicle (X_i is the i^{th} vehicle or a vehicle in front of it). X_i takes its values in $\{1, 2, .., i\}$. The vehicle with index X_i brakes after a reaction time τ because it receives the warning message from Veh_0. Consequently, Veh_i will brake after $(i + 1 - X_i)\tau$ seconds. If there is no equipped vehicle then we consider $X_i = 1$. As a vehicle is equipped with a radio with a probability p independently of other vehicles, X_i has the following distribution:

$$\begin{cases} \mathbb{P}(X_i = 1) = & (1-p)^{i-1} \\ \mathbb{P}(X_i = k) = & p(1-p)^{i-k} \quad \text{for } k \in \{2, .., i\} \end{cases} \quad (26)$$

Indeed, $\{X_i = 1\}$ if the $i - 1$ leading vehicles do not have a radio, and $\{X_i = k\}$ if the $i - k$ preceding vehicles do not have a radio whereas the k^{th} has one. The parameter p may also takes into account the proportion of drivers who do not brake when they receive the warning message. In this case, $p = p_e * p_b$ where p_e is the probability for the vehicle to be equipped and p_b is the probability for an equipped vehicle to brake. Such an assumption does not modify the demonstration and the results. The distance from the initial collision is $\sum_{k=1}^{X_i} d_{inter(k-1,k)} + X_i l$. Let $delay(d)$ be the delivery delay of the warning message for a vehicle at a distance d of the initial collision. The distance covered by the vehicle X_i between the collision and the reception of the safety message is then $v \times delay(\sum_{k=1}^{X_i} d_{inter(k-1,k)} + X_i l)$. Also, we have to consider the case where the message propagates very slowly, and where the accumulation of the reaction times is less than the delivery of the message. The distance covered by the vehicle X_i between the collision and the time the vehicle begins to react (after its own reaction time) is:

$$min\Big(v \times delay\left(\sum_{k=1}^{X_i} d_{inter(k-1,k)} + X_i l\right), (X_i - 1)d_\tau\Big) \quad (27)$$

By convenience, we set:

$$g(u) = min\Big(v \times delay\left(\sum_{k=1}^{u} d_{inter(k-1,k)} + ul\right), (u-1)d_\tau\Big) \quad (28)$$

Veh_i will crash if it cannot stop before its preceding vehicle:

$$crash(i) = \begin{cases} 1 & \text{if } g(X_i) + (i + 1 - X_i)d_\tau \\ & +d_{dec} > \sum_{k=1}^{i} d_{inter(k-1,k)} \\ 0 & \text{otherwise} \end{cases} \quad (29)$$

Let C be the random variable describing the number of collisions, C can be defined as:

$$C = \sum_{i=1}^{+\infty} crash(i) \quad (30)$$

We are interested in the computation of the mean value of C, denoted as $E[C]$. We get,

$$\begin{aligned} E\left[C\right] & \\ = & E\left[\sum_{i=1}^{+\infty} crash(i)\right] \\ = & \sum_{i=1}^{+\infty} \mathbb{P}\Big(g\left(X_i\right) + (i+1-X_i)d_\tau + d_{dec} \\ & > \sum_{k=1}^{i} d_{inter(k-1,k)}\Big) \end{aligned} \quad (31)$$

As the random variable $\sum_{k=1}^{X_i} d_{inter(k-1,k)}$ is included in the sum $\sum_{k=1}^{i} d_{inter(k-1,k)}$ (in Equation (31)), we split $\sum_{k=1}^{i} d_{inter(k-1,k)}$ in two subvariables: the sum from 1 to X_i and the sum from X_i+1 to i.

It is well known that the sum of k independent exponential variables with parameter λ follows a *Gamma* distribution with parameters $\left(k, \frac{1}{\lambda}\right)$. Therefore, $\sum_{k=1}^{X_i} d_{inter(k-1,k)}$ and $\sum_{k=X_i+1}^{i} d_{inter(k-1,k)}$ follow *Gamma* distributions with parameters $\left(X_i, \frac{1}{\lambda}\right)$ and $\left(i-X_i, \frac{1}{\lambda}\right)$. Moreover, the two sums are independent.

Equation (31) consists thus in computing

$$\begin{aligned} E\left[C\right] = \sum_{i=1}^{+\infty} \mathbb{P}\Big(& min\big(v \times delay\left(U_{X_i} + X_i l\right) \\ & , (X_i - 1)d_\tau\big) + (i+1-X_i)d_\tau + d_{dec} \\ & > U_{X_i} + V_{X_i}\Big) \end{aligned} \quad (32)$$

with $U_{X_i} \rightsquigarrow \Gamma\left(X_i, \frac{1}{\lambda}\right)$, $V_{X_i} \rightsquigarrow \Gamma\left(i-X_i, \frac{1}{\lambda}\right)$ and where the distribution of X_i is given by Equation (26).

D. Numerical results on the number of collisions.

In this Section, we present the numerical results for the mean number of collisions. They are obtained by numerical evaluation of Formula (32). The set of parameters are given in the bottom of Table I. For the function $delay(.)$ we use the approximation d_{approx} given by Formula (17), which has been proved to be the most accurate.

a) Number of collisions with regard to the capacity: In Figure 4(a), we plotted the mean number of collisions. We consider different penetration ratio of radio technology, from 0% to 100% of equipped vehicles. The mean capacity (see Equation (19)) varies from 1800 to 3200 veh/h (the mean number of vehicles varies from 14 to 28 per kilometer). The upper value corresponds to the case where the mean distance between two successive vehicles is equal to $d_\tau + d_{dec}$. It is the minimum distance to stop if the driver is instantaneously informed of the accident. Figure 4(a) shows that even for a penetration ratio of 1% there is a significant reduction in the number of collisions. A penetration ratio of 25% is sufficient to drastically reduce the number of collisions (from 105 to 6 collisions when the capacity is 3200 veh/km).

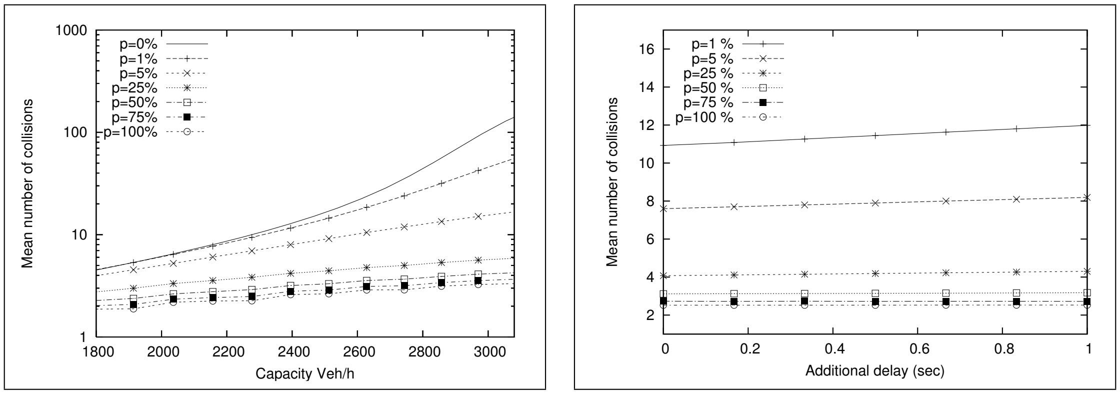
b) Impact of retransmission delay on the number of collisions: When a vehicle has to forward the safety message, this forwarding may be delayed. This delay may be caused by a full emission buffer or by a busy medium. To measure the impact of the forwarding time on the number of collisions, we add a constant (from 0 to 1 sec) to the function $timer(.)$. The results are shown in Figure 4(b) for a capacity of 2400 veh/h. This additional delay does not significantly increase the number of collisions. Indeed, vehicles in the radio range of the crashed car received the message directly (without multi-hop) and are thus not impacted by the forwarding delay. Vehicles at least one hop away from the initial collision perceived a higher delay in the reception of the message, but are unlikely to be involved in the accident (due to the distance from the accident). Consequently, even with an additional delay the probability of collision is low. We observe the same behavior for capacity up to 3200 veh/km.

We also performed the same study for two other *FER* functions (the boolean model and a *FER* function modeling Rayleigh propagation environment). The results on the number of collisions are not shown here because they are very similar to the ones obtained with the 2RM model.

V. Conclusion

In this paper we have proposed a probabilistic framework based on point process to evaluate the delivery delay of safety message in a string of vehicles. In the second part, we assessed the benefit of using IVC in reducing the number of collisions after an accident. We studied the impact of both communication devices penetration ratio and delivery delay on this number of collisions. It appears that, whatever the radio propagation environment, a penetration of approximately 25% is sufficient to reduce drastically the number of accidents. We have also shown that broadcast efficiency is relatively invariant to delays induced by forwarding safety messages.

This work may be extended in several ways. We plan to propose and study other point process to model other spatial distributions of vehicle locations. Cluster point process for instance, can be used to model situations where some vehicles (a truck for instance) slow down the vehicles behind. We also intend to analyze the influence



(a) Mean number of collisions for different penetration ratios.

(b) Impact of an additional delay in the message forwarding on the number of collisions.

Fig. 4. Mean number of collisions for different penetration ratio and different delay in the message forwarding.

of various parameters involved in vehicle movements. For the time being, we figure that a more complex modeling will not drastically alter our current results.

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