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Analysis and Simulation of a Broadcast protocol for VANET

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SUMMARY

This paper addresses the analytical evaluation of broadcast protocols in VANET. We focus on the most popular broadcast algorithm, which consists in the current emitter selecting the furthest receiver as the next forwarder. We propose a general framework based on point process to evaluate this protocol. The radio environment is modelled by a generic *Frame Error Rate* (*FER*) function. It enables us, through the same model, to assess the performances for different radio environments or radio technologies. We derive simple formulas for different quantities relative to the performance of the broadcast protocol: time to propagate the message, emitters' intensity, mean number of receptions for the same message, etc. Results based on the analytical model are compared to simulations using realistic vehicle traffic in order to determine the contexts for which the analytical model is relevant.

KEY WORDS: Inter-Vehicle Communications, VANET, Broadcast, Performance Evaluation.

1. Introduction

Vehicular Ad-hoc NETworks (VANETs) are a special kind of Mobile Ad-hoc NETwork (MANET). It assumes that all, or a subset of the vehicles is equipped with radio devices, enabling communications between them. Specific technologies like IEEE 802.11p [12] (also referred to as Wireless Access in Vehicular Environments, WAVE) are dedicated to enabling these vehicle-to-vehicle radio communications. One of the major applications of VANET aims to increase vehicle safety and driver convenience. These kind of applications rely on the dissemination of warning and control information [18]. This allows a vehicle to obtain information about accidents, congestion and the surface conditions of roads from other vehicles. The performances of the broadcast algorithm used to disseminate the messages in the VANET are thus crucial. Due to the particular topology, the particular mobility of the vehicles and the special requirements of the applications, broadcast algorithms in VANET are different from the ones used for the classical MANET. Therefore, the broadcast protocols must be evaluated in this particular environment. Some analytical works exist on the performance evaluation of VANET, describing their structural properties such as connectivity, route lifetime and capacity [19, 29, 17, 20]. But there are only a few analytical studies about the performance of broadcast protocols. Indeed, most of the studies use simulations ([7, 26, 2] to cite a few). In [28], the effects of broadcast flooding and several schemes to reduce redundant broadcasts in Ad-Hoc networks are analyzed. [4] proposes a model to assess the overhead, coverage and latency characteristics of a particular broadcast algorithm for VANET. The model uses simplistic radio assumptions, where the radio range of the nodes is fixed and the same for all the nodes. Moreover, the algorithm considered is very simple and not realistic in the context of VANET. The article closely to this paper is [9], where the authors develop an analytical framework to study broadcast performances and derive several metrics relevant to the dissemination of safety messages. There are two limitations with their model: a discrete space is used to represent vehicle locations, and they suppose an ideal radio environment with a fixed radio range.

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1.1. Contributions

This paper addresses the analytical evaluation of a broadcast protocol in VANET. We shall focus on the most popular broadcast algorithm, presented in detail in Section 2. The analytical results are then compared to realistic traffic patterns. The aim is to estimate the impact of the assumptions made in the analytical model on real performances. The contributions of this paper are as follows:

- We propose a general framework based on point process to evaluate the broadcast protocol. Our approach is based on the theory of point processes and Palm Calculus [25, 5].
- The radio environment is modelled by a generic *Frame Error Rate* (*FER*) function (it gives the probability of losing a frame with respect to the distance). We can thus model very different radio propagation environments or radio technologies.
- We derive formulae and bounds for different quantities relative to the performance of the broadcast protocol: time to propagate the message, emitters' intensity, mean number of receptions of the same message, etc.
- We use a micro-simulator, generating realistic traffic under different traffic densities, to determine the contexts for which the analytical model is pertinent.

1.2. Organization

The remainder of this paper is organized as follows. Section 2 overviews broadcast protocols in VANET and presents the one evaluated in this paper. In Section 3, we analyze the proposed broadcast scheme. Section 4 presents the different radio models and parameters used in the numerical evaluations. The analytical results are compared to simulations (based on the same assumptions as the theoretical model) in Section 5. In Section 6, we present the micro-simulator and compare the analytical results to simulations obtained with realistic traffic. Finally, concluding remarks and comments are given in Section 7.

2. Broadcast protocols

A broadcast protocol aims to send the same message to all the nodes in the network. The simplest broadcast mechanism consists in a node broadcasting the message after the first reception. This mechanism has the advantage of being simple, but it creates the famous storm problem [27], also named broadcast flooding, as it generates a great number of retransmissions and receptions. The goal of an efficient broadcast protocol is to minimize the number of transmissions while maintaining a high probability of reception.

In [27], broadcast protocols are categorized with respect to the criteria used by a potential forwarder to cancel its own retransmission: distance to the emitter, number of receptions, the result of a random variable, node locations, etc. The latter supposes that the nodes are capable of knowing their geographical locations, via GPS for instance. A node forwards the broadcast message when the additional coverage is greater than a predefined threshold. This approach is shown to be the most efficient, as it eliminates the most redundant rebroadcasts without compromising reachability. This algorithm has been improved and adapted in the context of VANET. Most of the broadcast protocols favour the furthest nodes from the previous emitter as the next forwarder. It maximizes the coverage area and minimizes the number of redundant receptions. For instance, in [3, 9], a vehicle retransmits the message according to a certain probability. This probability increases with the distance from the emitter and thus farther nodes are likely to be selected as forwarders.

In [2, 8, 16], the furthest receiver is systematically the next forwarder, but the way it is selected differs from one protocol to another. In [8], each node is supposed to know its neighbourhood (IDs and locations of the vehicles in its radio range). A forwarder selects in its neighbourhood the furthest node in the broadcast direction. A field in the message indicates the ID of the node responsible for the next retransmissions. In [2] and [16], when receiving a frame, a node triggers a retransmission timer (a blackburst in [16]) with a duration decreasing with the distance from the emitter. As a result, the furthest node retransmits first. Upon receiving this broadcast, the other nodes cancel their own transmission.

In this paper, we consider an example of this approach. The algorithm is as follows. We study the propagation of the message in a given direction (upstream with regard to the flow of vehicles). When a node at y (y represents both the node and its location) receives for the first time the broadcast message from a node x with $x < y$, it triggers a timer. The initial value of the timer, denoted $timer(|y-x|)$, decreases with the distance from the emitter ($|y-x|$). If the node y does not receive the same message from a node z with $z > y$ at the timer expiration, it retransmits the message, otherwise it cancels its transmission. There is no acknowledgement from the receivers; therefore, the message dissemination can stop.

3. Analytical model

To model vehicle locations, we use a point process in $\mathbb{R}$. Basically, it consists of a sequence of points randomly distributed on the line, each point representing the location of a vehicle. For convenience sake, we shall use the same notation to represent a vehicle and its location. We consider a point process distributed on the line rather than in the plane ($\mathbb{R}^2$) as the radio scopes of the vehicles are really greater than the road width. The point process can model one or several lanes, for a highway for instance. The considered point process is then the superposition of a set of independent point processes, one for each lane. Our study applies to cases where all, or a proportion l ($0 < l \leq 1$), of the vehicles are equipped with radio devices. As the considered underlying process, modelling all the vehicles, is Poisson the thinning of the process representing the equipped vehicles is also Poisson. To take into account only a proportion l of equipped vehicles, we can simply to multiply the intensity of the process by l in all the formulae of the analytical study.

A transmission from a vehicle is properly received by a vehicle at distance d with probability $p(d)$. The function $p(.)$ is the *Frame Error Rate* (*FER*) with respect to the distance. This function takes its value in $[0, 1]$, and is supposed to be continuous and increasing with $p(0) = 0$ and $\int_0^{+\infty}(1 - p(x))dx < +\infty$. We use the same function $p(.)$ for the transmissions from all the vehicles and we suppose that receptions are independent between the vehicles.

We consider two different models. The first one allows us to assess the distance covered by the message. It is defined as the distance between the node which initiates the broadcast and the furthest node which receives it. The second one is used to estimate the intensity of the forwarders and the mean number of receptions per node. In this model, we suppose that forwarders form a stationary point process (their distribution does not change with distance). With this assumption, the cited quantities can be computed for a typical point, without taking into account the distance to the initial emitter. We also use this model to compute the delivery delay of the message.

3.1. First model: distance covered by the message.

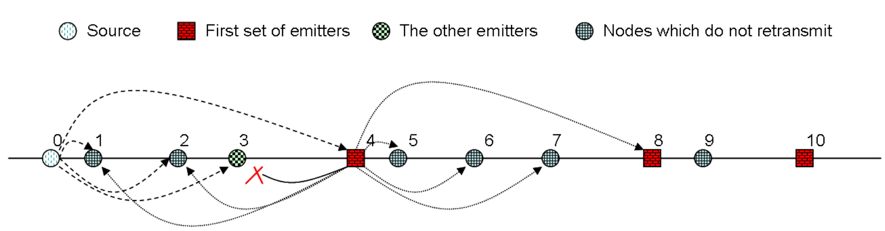


Figure 1. Example of a broadcast scenario.

Vehicle locations are modelled by a Poisson point process Φ with intensity λ (the mean number of nodes/vehicles per kilometer). Even if the Poisson point process is often considered to model vehicle locations, this assumption is only ascertained for low vehicle density [22, 21, 13, 24]. We discuss the relevance of the Poisson process for high intensities in detail in Section 6. In order to estimate the total distance covered by the message, we look at the points (the vehicles) involved in the progression of the message. According to the broadcast protocol, we can distinguish two kinds of emitters. This distinction is easier to understand with an example. The example is depicted in Figure 1. The node which initiates the message is the node 0. This first broadcast is received by nodes 1, 2, 3 and 4. Node 4, as it is the furthest from 0, is the first node to retransmit the message. The message is received by nodes 1, 2, 5, 6, 7 and 8. So, node 8 retransmits (not shown in the figure). Node 3 did not receive the retransmission from 4, therefore it retransmits the message (not shown in the figure) at the expiration of its timer. We can thus distinguish a first set of emitters contributing to the fast progression of the message (nodes 4 and 8 in our example), and the emitters which retransmit because they did not receive the message from nodes upstream (node 3 in our example). The first set of emitters is denoted $(T_i)_{i\geq 0}$ in this model.

Let $T_0 = 0$ be the location of the node which initiates the broadcast. We denote by T_1, the furthest node which received the message from T_0. As the furthest receiver is the next receiver, T_1 will retransmit the message. The furthest node which received the message from T_1 is denoted T_2, and so on. The sequence $(T_i)_{i\geq 0}$ is thus defined recursively. T_i is then formally defined as the furthest node which received the message from T_{i-1}. The sequence $(T_i)_{i\geq 0}$ corresponds to the fastest progression of the message. In a perfect world where vehicles have a perfect radio range R ($p(u) = 0$ for $u < R$ and $p(u) = 1$ otherwise), the nodes $(T_i)_i \geq 0$ are the only emitters. Indeed, a node of Φ which does not belong to (T_i) will receive the message from the downstream node T_i, and the retransmission from T_{i+1} will cancel its own transmission.

After the transmission from T_0, nodes which have received the messages (from T_0) are the results of an independent thinning of Φ (a subset of Φ, where each point is selected independently of the others). The point process Φ is then split into two independent processes denoted Φ_{T_0} and Φ'_{T_0}. Φ_{T_0} represents the points of Φ which receive the message from T_0, Φ'_{T_0} represents the other points. As the probability of selecting a point depends on its distance from the origin, the intensity of these two point processes is inhomogeneous, but, thanks to the properties of the Poisson point processes, they are still Poisson (an independent thinning of a Poisson process is Poisson). An inhomogeneous Poisson point process is characterized by its intensity measure. This measures the mean number of points as a function of the considered interval. The intensity measure of the process Φ_{T_0} is given as:

$$\Lambda(A) = \lambda \int_A (1 - p(r))\, dr$$

for $A \subset I\!R^+$.

T_1 is less than x ($x \in I\!R^+$) if and only if the inhomogeneous process is empty in $[x, +\infty)$. Therefore, the *cumulative distribution function* of T_1 is:

$$\mathbb{P}\left(T_1 \leq x\right) = e^{-\Lambda((x,+\infty))} \tag{1}$$

The *probability density function* (*pdf*) of T_1 is then given by:

$$f_{T_1}(x) = \lambda(1 - p(x))e^{-\Lambda((x,+\infty))} + q\delta_0 \tag{2}$$

The *pdf* is composed of a continuous term and an atom at 0 ($\delta_.$ is the Dirac measure) corresponding to the event that there is no receiver: $q = e^{-\Lambda((0,+\infty))}$.

The probability of getting a second forwarder ($T_2 > 0$) is different to $(1 - q)$. Indeed, after the first hop, there is likely to be a gap after T_1. For instance, with the Boolean model where the radio range R is fixed, there are no Φ points in $[T_1, T_1 + R]$.

For the second hop $T_2 - T_1$, we consider the points which do not receive the message from T_0 (i.e. the points of Φ'_{T_0}) but from T_1. It results in a thinning of Φ'_{T_0}. Each point x in Φ'_{T_0} is selected with probability $p(|x - T_1|)$. Since T_1 is independent of Φ'_{T_0}, conditionally to the random variable T_1, the selected points form an inhomogeneous Poisson process with intensity measure $\lambda \int_. p(x)(1 - p(|x - T_1|))dx$. The probability that there is no point x with $x > T_1$ which receives the message from T_1 given that $T_1 > 0$ is:

$$\begin{aligned} q_2 &= \mathbb{E}\left[e^{-\lambda \int_{T_1}^{+\infty} p(x)(1-p(x-T_1))dx} | T_1 > 0\right] \\ &= \mathbb{E}\left[e^{-\lambda \int_{0}^{+\infty} p(x+T_1)(1-p(x))dx} | T_1 > 0\right] \end{aligned}$$

We can also compute the *complementary cumulative distribution function* of $T_2 - T_1$ given that $T_1 > 0$:

$$\mathbb{P}\left(T_2 - T_1 > x | T_1 > 0\right) = 1 - \mathbb{E}\left[e^{-\lambda \int_{x}^{+\infty} p(u+T_1)(1-p(u))du} | T_1 > 0\right]$$

This quantity can easily be computed with the knowledge of the *pdf* of T_1 conditionally to $T_1 > 0$. This *pdf* is given in the next subection by formula (4).

If we suppose that the distributions of $(T_{i+1} - T_i)_{i\geq 0}$ are independently and identically distributed with the same distribution as $T_2 - T_1$, the probability of the message stopping after k hops is then $(1-q)(1-q_2)^{k-1}q_2$. We consider the distribution of $T_2 - T_1$ rather than $T_1 - T_0$ for the hop lengths because it takes into account the gap created after each hop. The mean distance covered by the message, denoted $\mathbb{E}[D]$, is then:

$$\mathbb{E}[D] = (1-q)a + \frac{a_2(1-q)(1-q_2)}{q_2} \tag{3}$$

where $a = \mathbb{E}[T_1|T_1 > 0]$ and $a_2 = \mathbb{E}[T_2 - T_1|T_2 - T_1 > 0, T_1 > 0]$.

Of course, the assumption of the independence of the sequence $(T_{i+1} - T_i)_{i\geq 0}$ does not hold. Nevertheless, this assumption drastically simplifies the computations and does not significantly bias the numerical evaluations, as will be shown in Section 5.

It is worth noting, that in the case of the Boolean model where R can be randomly distributed, there are rigorous analytical results about the mean value of D, or even about its distribution [14, 29, 30]. By analogy with queuing theory, D corresponds to the busy period of a $M/GI/+\infty$ queue. But our model is more general, as we use a generic *FER* function rather than a random radio range.

3.2. The second model: number of receptions and delivery delay.

For the other quantities (mean number of receptions per vehicle, mean delay, etc.), we build a stationary point process to model the sequence of vehicles which forwards the message (the points T_i in the previous paragraph). It allows us to compute these quantities (especially the mean number of receptions) for a typical point independently of the distance from the initial transmitter. The results obtained are thus pertinent for vehicles some hops away (at least one hop) from the vehicle which initiates the broadcast. In the numerical evaluations, these quantities rapidly converge to the analytical results as the distance to the initial transmitter increases. Also, the stationarity of the point process allows us to apply Palm calculus. It offers a mathematical framework to compute esperance of point process functionals conditionally to the presence of a point (of this point process) at a given location. This is an intuitive definition. A more formal introduction of Palm calculus can be found in [5, 25, 10]. In our case, it is particularly useful. As an example, when computing the mean number of receptions for a point, we will suppose that this point is at a given location. This assumption impacts the distribution of the point process. For instance, the distribution of the distance between location x and the location of the previous forwarder is not the same depending on whether there is a point/vehicle at x or not. The probability measure is thus different under Palm measure. The computation of esperances under Palm measure is denoted $\mathbb{E}_N^x[.]$, meaning (intuitively) that we compute this esperance conditionally to the presence of a point of N at location x.

We consider a new point process built from the previous point process $(T_i)_{i\geq 0}$. In the first model, we considered that the sequence $T_{i+1} - T_i$ could stop. The idea here is to consider a sequence (S_i) of points distributed in $\mathbb{R}$ such that $S_{i+1} - S_i$ is almost surely positive. The distribution of $S_{i+1} - S_i$ is obtained by normalization of the *pdf* of $T_1 - T_0$:

$$f_{S_{i+1}-S_i}(x) = \frac{1}{1-q}\lambda(1-p(x))e^{-\Lambda([x,+\infty))} \tag{4}$$

This process, denoted Φ_S, is thus stationary with intensity $\lambda_S = \frac{1}{\mathbb{E}[S_{i+1}-S_i]}$. A second stationary point process Φ_O models the other nodes. It is an independent Poisson point process with intensity $\lambda_O = \lambda - \lambda_S$. The global process describing all the nodes is thus a stationary point process $\Phi = \Phi_S \cup \Phi_O$ with intensity λ.

3.2.1. Intensity of the emitters The emitters can be divided in two sets. The first set corresponds to the emitters of Φ_S for which we have already computed the intensity (λ_S). The second set are the emitters of Φ_O, which correspond to the nodes that do not received the retransmission from the forwarders S_i downstream and thus rebroadcast the message (node 3 in Figure 1 for instance).

Let us suppose that the emitters of Φ_O forms a stationary point process. Its intensity, defined as the mean number of emitters of Φ_O in $[0, 1]$, is given by $\lambda_O \mathbb{E}^0_{\Phi_O}[\mathbb{1}_{0\text{is an emitter}}]$, where $\mathbb{1}_{condition}$ is the indicator function equals 1 if *condition* is true and 0 otherwise. $\mathbb{E}^0_{\Phi_O}[\mathbb{1}_{0\text{is an emitter}}]$ is the probability that the typical point of Φ_O located at 0 is an emitter. If we apply the Neveu exchange formula (see [5] page 21 formula (1.3.4) for instance), we get:

$$\lambda_O \mathbb{E}^0_{\Phi_O}\left[\mathbb{1}_{0 \text{ is an emitter}}\right] \quad = \quad \lambda_S \mathbb{E}^0_{\Phi_S}\left[\sum_{x_i \in \Phi_O \cap [0,S_1]} \mathbb{1}_{x_i \text{ is an emitter}}\right] \tag{5}$$

The Neveu exchange formula expresses the relationship between two Palm measures. Intuitively, formula (5) says that the intensity of the emitters in Φ_O is equal to the mean number of emitters of Φ_O between two typical points of Φ_S (one located at $S_0 = 0$ and the other one at S_1) multiplied by the intensity of Φ_S. For convenience, we set

$$A = \mathbb{E}^0_{\Phi_S}\left[\sum_{x_i \in \Phi_O \cap [0,S_1]} \mathbb{1}_{x_i \text{ is an emitter}}\right]$$

i.e. the mean number of emitters of Φ_0 in $[0, S_1]$ under Palm expectation. The global intensity of emitters λ_E is then:

$$\lambda_E = \lambda_S A + \lambda_S$$

Therefore, the problem boils down to the computation of A. Let x be a typical point of Φ_O in $[S_0 = 0, S_1]$. We suppose that x is an emitter if and only if:

- it receives the message from S_0,
- it does not receive the message from S_1,
- it does not receive the message from all the emitters of Φ_O in $[x, S_1]$.

The first emitter of Φ_O in $[S_0, S_1]$, denoted X_1, is the closest point from S_1 with $S_0 < X_1 < S_1$, which receives the message from S_0 but not from S_1. We set $X_1 = 0$ if there is no such point. Given S_1, the points which receive the message from S_0 and not from S_1 form an inhomogeneous Poisson point process with density measure $(1 - p(x))p(S_1 - x)$ with $x \in [S_0, S_1]$. The computation of the *pdf* of X_1 given S_1 is then straightforward:

$$\begin{aligned} f_{X_1|S_1}(x) \quad &= \quad (\lambda - \lambda_S)(1 - p(x))p(S_1 - x)e^{-(\lambda-\lambda_S)\int_x^{S_1}(1-p(u))p(S_1-u)du} \\ &+ \quad \left(1 - e^{-(\lambda-\lambda_S)\int_0^{S_1}(1-p(u))p(S_1-u)du}\right)\delta_0 \end{aligned} \tag{6}$$

where $1 - e^{-(\lambda-\lambda_S)\int_0^{S_1}(1-p(u))p(S_1-u)du}$ is the probability of having no emitter ($X_1 = 0$).

The second emitter X_2 of Φ_O (in $[S_0, X_1)$) is the closest point from X_1 with $S_0 < X_2 < X_1$ that receives the message from S_0, but not from S_1 and X_1. X_i is then the closest point of X_{i-1} such that $S_0 < X_i < X_{i-1}$ which receives the message from S_0, but not from the set of points $\{S_1, X_1, .., X_{i-1}\}$. The distribution of X_i becomes quickly intractable as i increases. Indeed, it depends on the random variables $\{S_1, X_1, .., X_{i-1}\}$. As a result, we build upper and lower bounds on A, leading to bounds on λ_E. We also propose an empirical approximation of this quantity. Due to lack of space, the proof is presented in appendix 7.

Proposition 1. *A first upper bound on A is:*

$$A_{upper} = \sum_{i=1}^{+\infty}\left(1 - \mathbb{E}^0_{\Phi_S}\left[e^{-(\lambda-\lambda_S)\int_0^{S_1}(1-p(x))p(S_1-x)^i dx}\right]\right)$$

A sharper upper bound is given by:

$$A_{upper2} = \left(1 - \mathbb{E}^0_{\Phi_S}\left[e^{-(\lambda-\lambda_S)\int_0^{S_1}(1-p(x))p(S_1-x)dx}\right]\right) + \sum_{i=2}^{+\infty}\left(1 - \mathbb{E}^0_{\Phi_S}\left[e^{-(\lambda-\lambda_S)\int_0^{X_1}(1-p(x))p(S_1-x)p(X_1-x)^{i-1}dx}\right]\right)$$

A straight lower bound consists in taking only the emitters in Φ_S. In other words, we consider $A = 0$.

The approximation is built as follows. Given the sequence $(x_j)_{j=0,..,i-1}$ with $x_0 = S_1$ and $0 < x_{i-1} < .. < x_2 < x_1 < x_0$, we compute x_i as a solution of the equation:

$$\int_{x_i}^{x_i-1}(1-p(x))\prod_{j=0}^{i-1}p(x_j-x)dx = 1$$

We stop when $x_i < 0$. Let I be the index of the smallest x_i such that $x_i > 0$. The mean number of emitters is then estimated as:

$$A_{approx} = I + \int_0^{x_I}(1-p(x))\prod_{j=0}^{I}p(x_j-x)dx$$

Remark 1. *The approximation consists in estimating the emitters' location in an iterative manner. The location of the first emitter x_1 is set in such a way that the mean number of points in $[x_1, S_1)$ which received the message from $S_0 = 0$ but not from S_1 is exactly 1. The location of the second emitter x_2 is set in such a way that the mean number of points in $[x_2, x_1)$ which receive the message from S_0 but not from S_1 and x_1 is 1, and so on. We stop when we reach 0 and count the number of "x_i" that we set.*

3.2.2. Mean number of receptions Let Φ_E be the stationary point process of intensity λ_E modelling all the emitters, the mean number of receptions is computed for a point added at 0:

$$\begin{aligned} R &= \mathbb{E}\left[\sum_{x_i\in\Phi_E}\mathbb{1}_{0 \text{ receives the message from } x_i}\right] \\ &= \lambda_E\int_{\mathbb{R}}(1-p(|x|))dx \end{aligned}$$

The second equality is obtained by applying the refined Campbell formula (for more on this formula, see [25] page 119). Bounds on this quantity are directly deduced from the bounds on λ_E.

3.2.3. Delay We estimate the delivery delay of the message for a node located at x ($x > 0$), and at distance x from the node which initiates the broadcast. We suppose that the message has been initially broadcasted by a node located at 0 at time $t = 0$. Let d_S be the mean delay of a retransmission, i.e. the mean delay between the reception and the retransmission for a node of Φ_S. We get,

$$d_S = \mathbb{E}^0_{\Phi_S}\left[timer(S_1)\right] + T$$

where $timer(.)$ is the function representing the duration of the timer with regard to the distance. T is a constant representing the average time to send the message, i.e. time to access the medium, send the frame, etc. The value of d_S is known since the distribution of S_1 under Palm expectation is given by formula (4). As for the intensity of the emitters, we propose in the next proposition a lower and an upper bound on this delay. The way they are built is given in the appendix.

Proposition 2. *We propose d_{min} as a lower bound on the delay at x ($x > 0$), with:*

$$d_{min} = d_S\left(max(\lambda_S x - 1, 0) + \mathbb{E}^0_{\Phi_O}\left[p(S^-(0))\right]\right)$$

We propose d_{max} as an upper bound on the delay at x, with:

$$d_{max} = d_S \left(\lambda_S x + \mathbb{E}^0_{\Phi_O}\left[p(S^-(0))\right]\right) + (d_0 - d_S)\,\mathbb{E}^0_{\Phi_O}\left[p(S^-(0))p(S^+(0))\right]$$

An approximation on the delay is given by:

$$d_{approx} = d_S \left(max(\lambda_S x - 0.5, 0) + \mathbb{E}^0_{\Phi_O}\left[p(S^-(0))\right]\right) + (d_0 - d_S)\,\mathbb{E}^0_{\Phi_O}\left[p(S^-(0))p(S^+(0))\right]$$

with

$$\begin{aligned}\mathbb{E}^0_{\Phi_O}\left[p(S^-(0))\right] &= \lambda_S \mathbb{E}^0_{\Phi_S}\left[\int_0^{S_1} p(x)dx\right] \\ &= \lambda_S \int_0^{+\infty}\int_0^{y} p(x)dx f_{S_1}(y)dy\end{aligned}$$

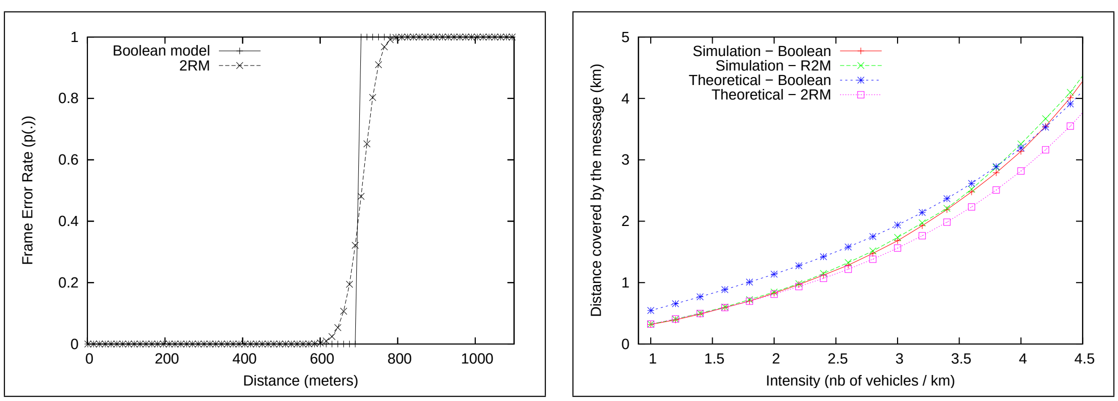
and

$$\begin{aligned}\mathbb{E}^0_{\Phi_O}\left[p(S^-(0))p(S^+(0)\right] &= \lambda_S \mathbb{E}^0_{\Phi_S}\left[\int_0^{S_1} p(x)p(S_1 - x)dx\right] \\ &= \lambda_S \int_0^{+\infty}\int_0^{y} p(x)p(y-x)dx f_{S_1}(y)dy\end{aligned}$$

4. Radio models and retransmission timer.

Simulation Parameters	Numerical values	Simulation parameters	Numerical values
Frequency	5.9GHz	Number of samples (simulation)	60000
Transmission Rate	3 Mb/s	Size of the observation window	from 0 to 25 km
Antenna heights	1.5 meters	Message length	100 bytes

Table I. Parameters.



(a) *FER* functions (b) Distance covered by the message

Figure 2. *FER* functions and distance covered by the message.

4.1. Radio models: Frame Error Rate

We consider two different *FER* functions, which are presented in the following paragraphs. They are set according to the IEEE 802.11p standard.

2RM model In order to set a realistic function $p(.)$, we use the *FER* model developed in [6]. In this paper, the authors presented a measurement-based model of the frame error process in a rural environment. The proposed model is based on the two-ray path loss model, hereafter referred to as 2RM. The model takes into account wavelength of the 802.11p standard, heights, distances and gains of the two antennas (emitter and receiver), frame length, etc. Using the default parameters of the 802.11p standard, we obtain the *FER* plot in Figure 2(a). Parameters are listed in table I. The radio range obtained with this model corresponds to the expected radio range of 802.11p, which is supposed to vary from 600 meters to 1 km.

Boolean model We also consider the classical Boolean model where the nodes have a fixed radio range. The *FER* $p(x)$ is then equal to 0 for x in $[0, R]$ and 1 for $x > R$. R is set to 700 meters in order to get radio range comparable with the 2RM model.

4.2. Retransmission timer

The function $timer(.)$ must decrease with the distance. We choose a function decreasing linearly with the distance, and where the timer is at most $1000\mu s$:

$$timer(x) = (-ax + b)1000$$

With the chosen radio models, the maximum distance between the emitter and the receiver is 1100 meters, so we get $a = \frac{1}{1100}$ and $b = 1$. We add to this delay the time T required by a forwarder to access the channel and transmit its frame. The MAC layer in 802.11p is similar to the IEEE 802.11e Quality of Service extension. Application messages are categorized into one of the four queues depending on their priorities. During the selection of a packet for transmission, the four queues contend internally. The selected packet then contends for the channel externally using its selected contention parameters. We consider here that safety messages use the highest priority. For the highest priority, a frame must wait $AIFS = 2t_s$ (Arbitration Inter-Frame Space) where t_s is the slot time ($t_s = 16\ \mu s$). Next, the transmission waits for a contention period randomly selected in the Contention Window (CW), where $CW = [0, 3t_s]$. So, it will be equal to $\frac{3}{2}t_s$ on average. Here, we suppose that a forwarder systematically wins access to the channel, as it uses the highest priority and the $timer(.)$ function. The value of T is then $T = \frac{7}{2}t_s + 267 = 323\ \mu s$, where 267 μs is the time to transmit a frame of 100 bytes at 3 Mbit/s. Therefore, the mean delay d_S of a forwarder in Φ_S, is

$$\begin{aligned} d_S &= \mathbb{E}[-a(S_{i+1} - S_{S_i}) + b]1000 + T \\ &= \left(\frac{-a}{\lambda_S} + b\right)1000 + T \end{aligned}$$

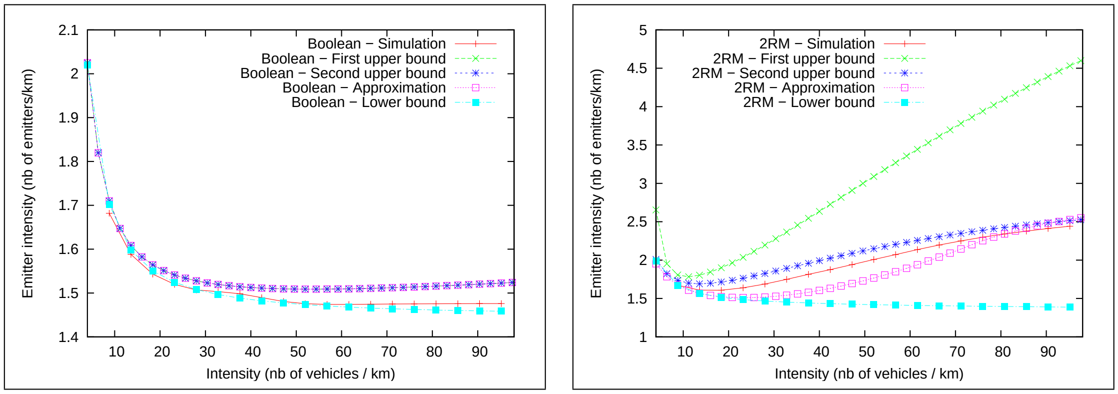
and the maximum delay for a forwarder is given by $d_0 = timer(0) + T = 1323\mu s$.

5. Model Evaluation

In this Section, we compare the different analytical results to simulations. We use a simulator coded in C, which implements the studied broadcast algorithm. It sets the vehicles's location according to a Poisson point process. The model used by this simulator is then very close to the theoretical one. It is used to estimate the accuracy of the bounds of propositions 1 and 2 in relation to the real values, and to observe the impact of the assumptions made in the different estimations (independence of $T_i - T_{i-1}$ in formula (3), stationarity of Φ_S and the assumption made in Proposition 2).

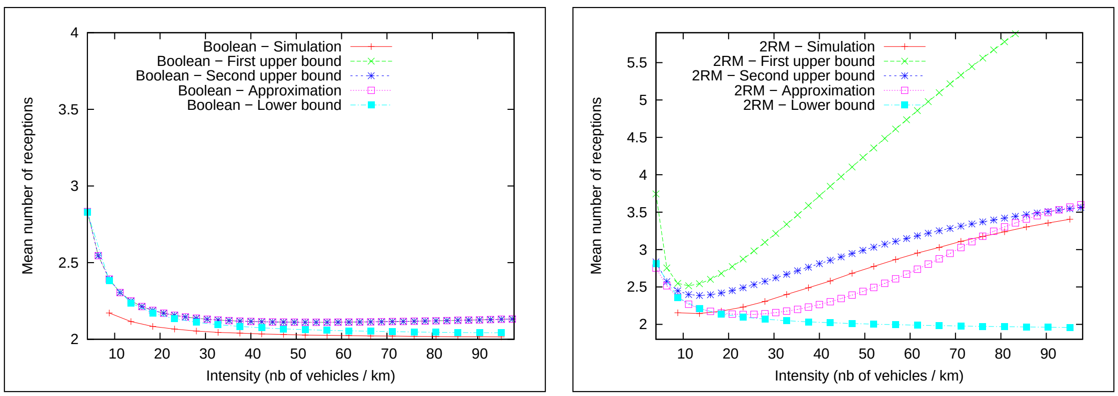
The set of parameters are given in Table I. The confidence intervals are not represented in the figures because they are too small due to the great number of samples.

Distance covered by the message The total distance covered by the message is presented in Figure 2(b). We chose small intensities of vehicles (between 1.0 and 5 veh/km) because for higher intensities the values soar. There is a very small gap between theoretical formulae and the average obtained by simulations.



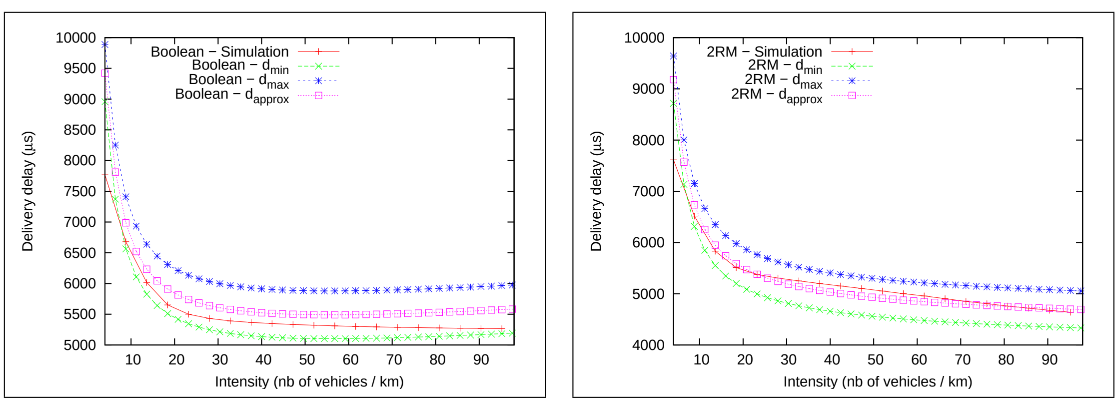
(a) Boolean. (b) 2RM.

Figure 3. Intensity of the emitters for the different *FERs*.



(a) Boolean. (b) 2RM.

Figure 4. Mean number of receptions per node for the different *FERs*.



(a) Boolean. (b) 2RM.

Figure 5. Delivery delay for a node at 5 km from the first emitter for the different *FERs*.

Intensity of the emitters For the quantities relative to the second model, we vary the intensity from 8 to 100 vehicles per kilometer. It makes no sense to consider lower intensities, since it models the dissemination of the message under stationary assumptions. Therefore, the theoretical evaluations are not accurate for the first point (intensity=8.0) in certain curves. In Figure 3(a) to 3(b), we plotted the intensity of the emitters obtained by simulation and compare it to the theoretical bounds given in Section 3.2.1. For the Boolean model, all of the theoretical evaluations lead to the value of λ_S, since the points of Φ_S are the only emitters. For the other *FER* function, the second upper bound and the approximation offer the best evaluations. We observe that the intensity of the emitters is higher when the *FER* is 2RM.

As we already mentioned, the only emitters with the Boolean model are the points of Φ_S. When the *FER* is 2RM, we observed from the samples that we can have at most one emitter just after each point of Φ_S corresponding to a node x which receives the message from the point $S^-(x)$ downstream, but not from the point $S^+(x)$ upstream.

Mean number of receptions The mean numbers of receptions per node, plotted in Figures 4(a) and 4(b), copy the behavior of the emitters' intensity. Estimations from the second upper bound and the approximation lead to very accurate results. Since the number of emitters is greater for 2RM, we logically get a higher number of receptions per node.

Delivery delay In Figures 5(a) to 5(b), we plotted the delivery delay for a vehicle 5 km away from the source of the message. For the two *FER* functions, the delay is perfectly bound by the analytical formulae.

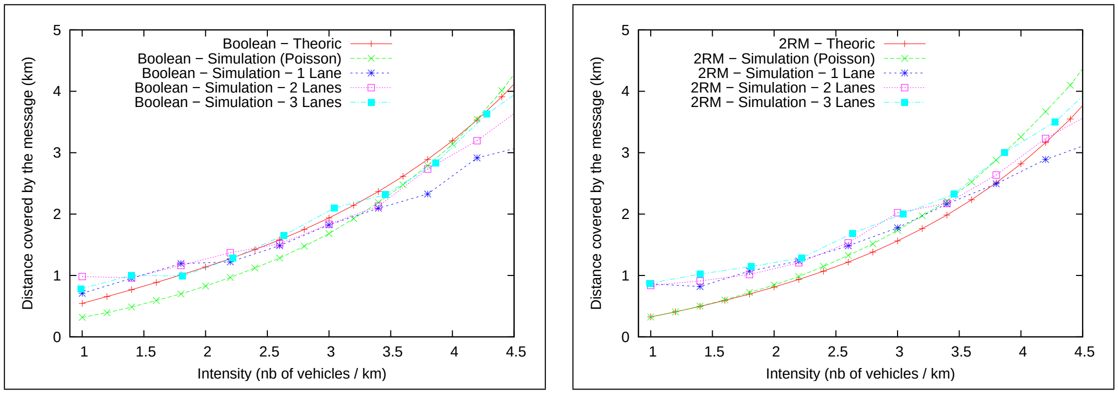
6. Traffic simulator

As aforementioned, for small densities of vehicles, vehicle locations follow a Poisson point process. For greater densities, the driver's behavior (braking/accelerating) depends on the other vehicles. In this Section, we assess the error in the performance of the broadcast algorithm when a Poisson process with high intensity is used. We propose the use of micro-simulations [11] to generate realistic traffic of vehicles. We evaluate the broadcast protocol with this traffic simulator and compare the results to the theoretical ones. The broadcast protocols and the FER functions are the same as in the previous Section, but vehicle trajectories are obtained from the traffic simulator rather than from the Poisson point process.

6.1. Presentation

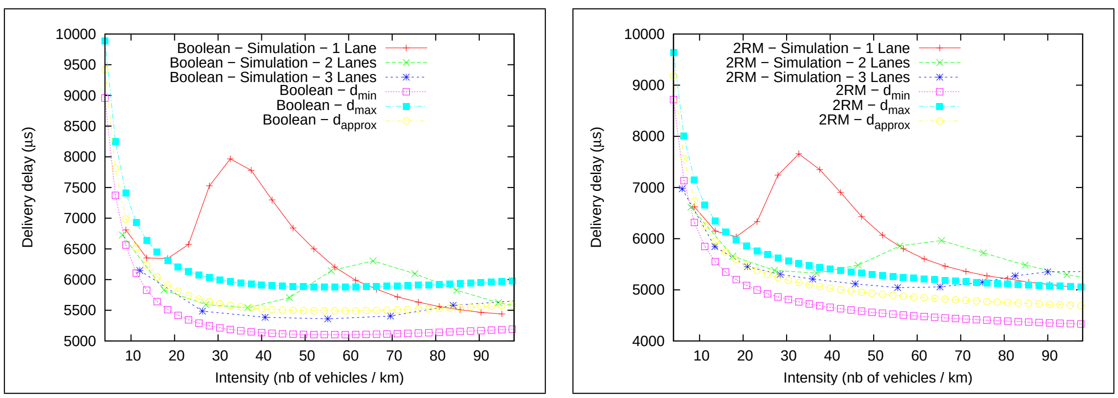
In order to obtain vehicle movements close to reality we have developed a traffic simulator. This traffic simulator allows us to faithfully emulate driver behavior. On a highway, driver behavior is limited to accelerating, braking or changing lanes. We assume that there is no off-ramp on the section of highway. A desired speed is associated to each vehicle. It corresponds to the speed that the driver would reach if he was alone in his lane. If the driver is alone (the downstream vehicle is sufficiently far), he adapts his acceleration to reach his desired speed (free flow regime). If he is not alone, he adapts his acceleration to the vehicles around (car following regime). He can also change lanes if the conditions of another lane seem better. All these decisions are functions of environment of the vehicles (speed and distance) and random variables used to introduce a different behavior for each vehicle. This kind of simulation is called micro simulation and the model we used is presented in detail in [1]. The model has been tuned and validated with regard to real traces observed on a highway.

We simulated a road/highway of 14 km with 1, 2 and 3 lanes. The desired speed of vehicles follows a Normal distribution with mean 120 km/h and standard deviation $\sigma = 10$. We used the same densities of vehicles as in the previous sections. The vehicles' density shown in the Figures corresponds to the mean number of vehicles entering at the beginning of the simulated highway. When we considered several lanes, the density was divided by the number of lanes. The abscissa in the figures is then the sum of the intensities on the different lanes.



(a) Boolean. (b) 2RM.

Figure 6. Comparison of the distance covered by the message with the micro-simulator and the Poisson process.



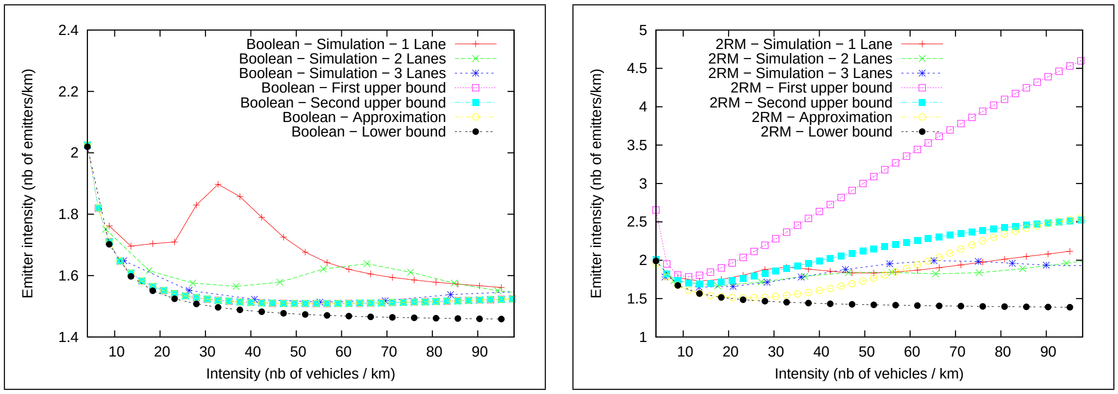
(a) Boolean. (b) 2RM.

Figure 7. Comparison of the delivery delay for a node at 5 km with the micro-simulator and the Poisson process.

We compare results obtained with the micro-simulator and the theoretical results.

Distance covered by the message We plotted the distance covered by the message in Figures 6(a) and 6(b). The errors between the theoretical results and the results obtained with the micro-simulator are less than 1 km.

Delay In Figures 7(a) and 7(b) we plotted the delay with regard to the density of vehicles. For one lane, there is clearly a spike in the mean delay. This spike is caused by an inhomogeneous distribution of the vehicles on the road: very dense sections of the road/highway are followed by very sparse sections. We discuss this phenomena in detail in Section 6.3. Sections of road/highway with sparse intensities of vehicles increase the delay, which explains the observed results. For 2 and 3 lanes, this phenomena is reduced and shifted to higher intensities.



(a) Boolean.

(b) 2RM.

Figure 8. Comparison of the emitters intensity for the different *FER* with the micro-simulator and the Poisson process.

Intensity of the Emitters Results are shown in Figures 8(a) and 8(b). Simulations based on the micro-simulator and the Poisson process are similar when the *FER* is 2RM. For the boolean model, there is a significant difference, especially for 1 lane.

6.3. Discussion

When the traffic reaches a certain intensity, most of the vehicles adapt their speed with regard to their environment (the other vehicles). It is known that under high vehicle intensities [23, 15, 21], the traffic can be described in terms of different congestion phases: phases where the speeds of the vehicles are low and vary quite a lot between vehicles, and phases where the speed is lower and more equal. This phenomena explains the results obtained with the micro-simulator. When the intensity increases, the traffic goes through the different phases. The spikes in the curves correspond to a phase where temporary jams occur (very dense sections with low speeds). It may just be caused by a vehicle slowing down, generating a wave effect upstream. A very sparse section of the highway then follows this jam. This phenomena is often refered as *stop-and-go* traffic. When we observe vehicle densities on the simulated highway, we observe this phenomena. When the density of vehicles entering in the simulator is high, we find sections of the highway with a lot of vehicles (up to $4-5$ times the supposed density) corresponding to a jam, and following sections with only a few vehicles. As the formulae are applied to the global density (computed on the whole highway) and not to local densities, it causes the spikes in the figures.

The traffic is Poisson only for intensities less than 8 veh/km. Nevertheless, the simulations show that the use of the Poisson point process gives good approximations, except for intensities corresponding to the phase where the distribution of the vehicles is very inhomogeneous on the road/highway (*stop-and-go* traffic). Intensities for which this phase occurs depend on the number of lanes. The impact of this phase seems to be more significant for 1 than for 2 and 3 lanes, and varies with regard to the observed quantities and radio models.

7. Conclusion

This paper proposed a probabilistic framework to assess the performances of broadcast protocols in VANET. Our approach is based on point process representing the vehicle locations. One of the benefits of our model is it does not suppose a particular radio environment. It allows us to study the impact of *FER* on the broadcast algorithm. It appeared that the classical Boolean model gives the better results in terms of the number of redundant receptions. On the other hand, more realistic *FER* functions involve a greater number of redundant messages.The comparison of the analytical results with simulations using realistic traffic patterns shows that there is a phase of traffic for which the Poisson point process is particularly inaccurate.

In future works, we plan to study point processes able to model the traffic during the critical phase. We think that Cluster or ON-OFF point processes may be more suitable for modelling vehicle locations when the intensity along the road/highway is strongly inhomogeneous. Another avenue would might be to analyze different broadcast algorithms and study the impact of the different traffic phases on their performances.

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APPENDIX

Proof of Proposition 1

Upper bounds The upper bound is built as follows. Let us consider the potential emitters of Φ_O in $[S_0 = 0, S_1]$. The broadcast from S_1 thins a potential emitter at x with probability $p(S_1 - x)$. The i^{th} broadcast thins a potential emitter at x with probability $p(X_{i-1} - x)$. The upper bound consists in considering that all the broadcasts are performed by S_1. Given S_1, the point process describing the potential emitters after the i^{th} broadcast is then an inhomogeneous Poisson point process with density measure $(\lambda - \lambda_S)(1 - p(x))p(S_1 - x)^i$. There is a i^{th} emitter if this point process is not empty in $[S_0, S_1]$. It happens with probability

$$1 - e^{-(\lambda-\lambda_S)\int_0^{S_1}(1-p(x))p(S_1-x)^i dx}$$

The upper bound on the number of emitters of Φ_O in $[S_0, S_1]$ is then

$$A_{upper} = \sum_{i=1}^{+\infty}\left(1 - \mathbb{E}^0_{\Phi_S}\left[e^{-(\lambda-\lambda_S)\int_0^{S_1}(1-p(x))p(S_1-x)^i dx}\right]\right)$$

This bound may be rough, but we can easily refine it if we suppose that after the retransmission from S_1 the other retransmissions are performed by X_1. The density of X_1 is given by formula (6). We get :

$$A_{upper2} = \left(1 - \mathbb{E}^0_{\Phi_S}\left[e^{-(\lambda-\lambda_S)\int_0^{S_1}(1-p(x))p(S_1-x)dx}\right]\right) + \sum_{i=2}^{+\infty}\left(1 - \mathbb{E}^0_{\Phi_S}\left[e^{-(\lambda-\lambda_S)\int_0^{X_1}(1-p(x))p(S_1-x)p(X_1-x)^{i-1}dx}\right]\right)$$

Lower bound A straight lower bound consists in taking only the emitters in Φ_S. In other word, we consider $A = 0$, and we get $\lambda_E \geq \lambda_S$.

Approximation We propose a method to estimate the mean number of emitters in $[0, S_1]$. The method is as follows. We estimate the location x_1 with $x_1 < S_1$ such that there is exactly one potential emitter between x_1 and S_1 on average. We set the location of the first emitter at x_1, look for the location x_2 of the second emitter, and so on, until we reach 0. More formally, given the sequence $(x_j)_{j=0,..,i-1}$ with $x_0 = S_1$ and $0 < x_{i-1} < .. < x_2 < x_1 < x_0$, we compute x_i as a solution of the equation:

$$\int_{x_i}^{x_i-1}(1 - p(x))\prod_{j=0}^{i-1}p(x_j - x)dx = 1$$

We stop when $x_i < 0$. Let I be the index of the smallest x_i such that $x_i > 0$. The mean number of emitters is then estimated as:

$$A_{approx} = I + \int_0^{x_I}(1 - p(x))\prod_{j=0}^{I}p(x_j - x)dx$$

Remark 2. *In practice, we need to condition by the value of the random variable S_1. In the numerical evaluation, we use a Riemann integration with an interval of* 1 *meter. Computation is then very fast, less than* 1 *second with a typical computer.*

APPENDIX

Proof of Proposition 2

Lower bound on the delay (d_{min}) In the best case, the node at x receives the message from $S^-(x)$ directly, where $S^-(x)$ is the closest node of Φ_S from x with $S^-(x) < x$. If a_x is the mean number of emitters in $\Phi_S \cap [0, x]$ and d_S the mean delay added by each transmitter, the mean delay at x denoted $delay(x)$ is then $(a_x - 1)d_S$ ($a_x \geq 1$ as it systematically counts the first emitter at 0). However, the initial emitter at 0 does not add any delay, so we subtract 1 from a_x. The computation of a_x is not trivial. It is formally defined as:

$$a_x = \mathbb{E}^0_{\Phi_S}\left[\sum_{x_i \in \Phi_S} \mathbb{1}_{x_i \in [0,x]}\right]$$

Here, we use $max(\lambda_S x - 1, 0)$ as a lower bound of $a_x - 1$.

We add to this bound a term taking into account the fact that the node at x does not receive the message from $S^-(x)$, due to a frame error, but from $S^+(x)$ (the closest node of Φ_S from x with $S^+(x) > x$). It adds d_S to the delay on average. It is still a lower bound since we do not take into account the fact that a transmission may be received from an emitter of Φ_O with a greater delay. Formally, the probability of x receiving the message from $S^-(x)$ is given by:

$$\mathbb{E}^0_{\Phi_S}\left[(1 - p(x - S^-(x)))\right] \tag{7}$$

It may be estimated by the probability of a typical node of Φ_O receiving the message from the previous forwarder $S^-(0)$:

$$\mathbb{E}^0_{\Phi_O}\left[(1 - p(S^-(0)))\right] \tag{8}$$

We get,

$$\begin{aligned} delay(x) &\geq d_S a_x \mathbb{E}^0_{\Phi_O}\left[1 - p(S^-(0))\right] + d_S(a_x+1)\mathbb{E}^0_{\Phi_O}\left[p(S^-(0))\right] \\ &\geq d_S\left(a_x + \mathbb{E}^0_{\Phi_O}\left[p(S^-(0))\right]\right) \\ &\geq d_S\left(max(\lambda_S x - 1, 0) + \mathbb{E}^0_{\Phi_O}\left[p(S^-(0))\right]\right) \end{aligned}$$

From the Neveu's exchange formula of two Palm measures, we get:

$$\begin{aligned} \mathbb{E}^0_{\Phi_O}\left[p(S^-(0))\right] &= \lambda_S \mathbb{E}^0_{\Phi_S}\left[\int_0^{S_1} p(x)dx\right] \\ &= \lambda_S \int_0^{+\infty}\int_0^y p(x)dx f_{S_1}(y)dy \end{aligned}$$

Upper bound on the delay (d_{max}) The lower bound supposed that, in the worst case, x receives the message from $S^+(x)$. However, it may receive it from another node instead (different from $S^+(x)$ and $S^-(x)$). In the worst case, the delay generated by the last transmitter is $d_0 = timer(0) + T$. There are thus 3 possibilities for the first reception of node x:

- x receives the frame from $S^-(x)$ (with probability estimated as $\mathbb{E}^0_{\Phi_O}\left[1 - p(S^-(0))\right]$), the delay is then $a_x d_S$;
- x receives the frame from $S^+(x)$ (with probability estimated as $\mathbb{E}^0_{\Phi_O}\left[p(S^-(0))(1 - p(S^+(0)))\right]$), the delay is then $(a_x + 1)d_S$;
- x receives the frame from an emitter in the interval $(S^-(x), S^+(x))$ (with a probability estimated as $\mathbb{E}^0_{\Phi_O}\left[p(S^-(0))p(S^+(0))\right]$), an upper bound on the delay is then $a_x d_S + d_0$.

Here, we use $\lambda_S x$ as an upper bound on $a_x - 1$. The upper bound is thus,

$$\begin{aligned} delay(x) &\leq d_S a_x \mathbb{E}^0_{\Phi_O}\left[1 - p(S^-(0))\right] + d_S(a_x+1)\mathbb{E}^0_{\Phi_O}\left[p(S^-(0))(1 - p(S^+(0)))\right] + (a_x d_S + d_0)\mathbb{E}^0_{\Phi_O}\left[p(S^-(0))p(S^+(0))\right] \\ &\leq d_S\left(a_x + \mathbb{E}^0_{\Phi_O}\left[p(S^-(0))\right]\right) + (d_0 - d_S)\,\mathbb{E}^0_{\Phi_O}\left[p(S^-(0))p(S^+(0))\right] \\ &\leq d_S\left(\lambda_S x + \mathbb{E}^0_{\Phi_O}\left[p(S^-(0))\right]\right) + (d_0 - d_S)\,\mathbb{E}^0_{\Phi}\left[p(S^-(0))p(S^+(0))\right] \end{aligned} \tag{9}$$

with

$$\begin{aligned} \mathbb{E}^0_{\Phi_O}\left[p(S^-(0))p(S^+(0)\right] &= \lambda_S \mathbb{E}^0_{\Phi_S}\left[\int_0^{S_1} p(x)p(S_1 - x)dx\right] \\ &= \lambda_S \int_0^{+\infty}\int_0^y p(x)p(y-x)dx f_{S_1}(y)dy \end{aligned}$$

Approximation on the delay (d_{approx}) We also propose an approximation on the delay, which is not proved to be a lower or an upper bound. It consists in approximating $a_x - 1$ by $max(\lambda_S x - 0.5, 0)$ rather than $\lambda_S x$ in formula (9).