An overview of results on ad hoc network performances using spatial models

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***Abstract*—We propose a short overview of models and results based on spatial models used to evaluate the performance of ad hoc networks. Locations of nodes are distributed in a two dimensional area according to a stochastic point process, which allows us to obtain the averages and distributions of different performance quantities. It is particularly suited in the context of ad hoc networks as the topology has an important impact on performance. The different models presented are related to the radio properties, connectivity and capacity of ad hoc networks.**

***Index Terms*—Spatial model, point processes, ad hoc networks.**

I. Introduction

Wireless networks and communication have witenessed phenomenal growth in recent years. It has been one of the fastest growing segments of the communications industry, surpassing wired communications in many domains. The performance evaluation of these networks is thus fundamental. While geographical aspects of wired networks do not play an important role in performance, the location of the nodes has a great impact in ad hoc networks. For instance, if the density of nodes is small, interference should be small as there are only a few emitters, increasing the radio scope of the nodes. On the other hand, a longer distance between the nodes should limit the connectivity. Moreover, even for small intensity, interference may be high if a set of emitting nodes are gathered in the region where we measure interference. All these phenomena are thus difficult to understand as they depends greatly on the spatial distribution of nodes. Static topologies, such as grids, and simulations that take into account a finite set of topologies are inaccurate. They consider only specific patterns and do not garantee that the results obtained hold for other patterns. Stochastic point processes are thus particularly suited to the performance evaluation of ad hoc networks. They allow us to obtain averages and distributions for different quantities related to the performance of the networks. These statistical quantities are based on an infinite number of topologies (the samples). Another benefit is in describing statistical geographical properties with a few parameters (for example, we use only one parameter for the Poisson point process), leading to simpler interpretations of the results. It is worth noting that the results obtained for ad hoc networks may also be applied, in some cases, to more general wireless networks [16].

In this paper, we propose a short overview of models and results based on spatial models. We first present, in Section II, two point processes. The first one is the classical Poisson point process and the second is the Matèrn point process. In Section III, we study a link model, for which link properties (interference, *SINR*, *FER*, etc.) are calculated. In the next two sections, Sections IV and V, we sketch the main results for connectivity and capacity in ad hoc networks. We conclude in Section VI.

II. Point Processes

In this Section, we present two examples of point processes which can be used to model locations of the nodes of an ad hoc network. We are interested in point processes distributed in $I\!R^2$. We begin with the most commonly used point process, the Poisson point process. It has been used extensively in the modeling of ad hoc networks, to model interference and radio properties [3]–[5], [17], [26], to study the connectivity or capacity of ad hoc networks [8], [9], [15], etc. One of the definitions of the homogeneous Poisson Point process is as follows:

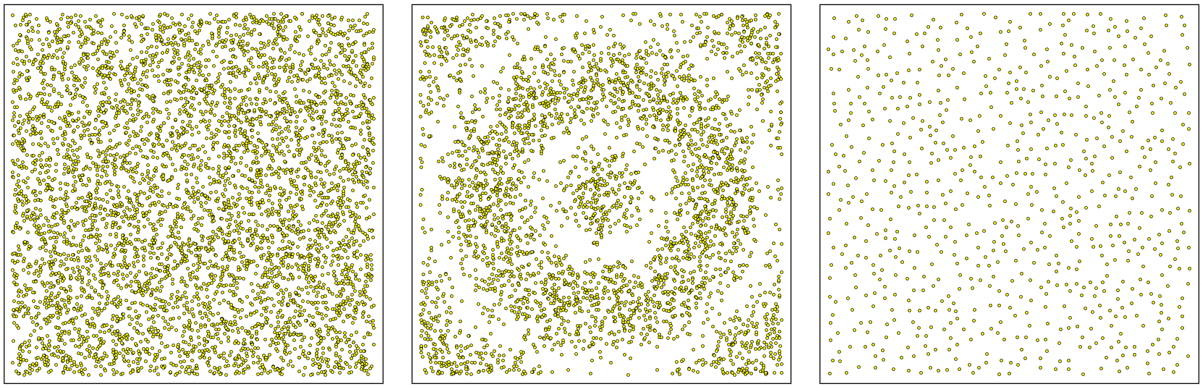
Definition 1. *A homogeneous Poisson point process Φ of intensity λ is characterized by two properties. They are:*

- *The number of points of Φ in a bounded Borel set B has a Poisson distribution of mean $\lambda|B|$, where $|B|$ is the Lebesgue measure of B in $I\!R^2$.*
- *The numbers of points of Φ in k disjoint borel sets form k independent random variables.*

A sample of a Poisson point process is shown in Figure 1(a). We can also consider inhomogeneous Poisson point process. As the name indicates, the mean number of points in a given area depends on the location of this area. More precisely, the definition of the inhomogeneous Poisson point process is the same as definition 1, except that the first assertion is changed to

- *the number of points in a Borel set B has a Poisson distribution of mean $\Lambda(B)$, where Λ is an intensity measure.*

In Figure 1(b), we draw a sample of such a process with the intensity measure



(a) Homogeneous Poisson point process.

(b) Inhomogeneous Poisson point process.

(c) Matèrn point process.

Fig. 1. Samples of point processes.

$$\Lambda(B) = \int_B cos(\|x\|)dx$$

While the Poisson point process is suitable for modeling all the nodes of an ad hoc network, and can thus be used to evaluate the capacity, connectivity and performances of routing protocols, it should not be used systematically to study radio properties, such as interference, Signal to Interference-plus-Noise Ratio (*SINR*), Bit Error Rate (*BER*), etc. Indeed, all these quantities depend on interference, and interference at a given time does not depend on all the nodes but only on the emitter locations. The Poisson point process is not always suitable for modeling these emitters, as it supposes, in some way, that they are independently distributed. However, in practice, most of the radio technologies (802.11, 802.15.4, etc.) use CSMA/CA medium acess protocol, which consists, for a potential emitter, in listening to the channel before emitting. If the interference level is greater than a given threshold, the channel is presumed busy and the transmission is delayed. Otherwise the emitter transmits its frame. This mechanism leads to a distribution of emitters that is more correlated than Poisson point processes.

An example of a point process which captures this phenomenon is the Matèrn point process. It was originally presented in [20]. A more accessible presentation of this point process can also be found in [25]. It belongs to the family of hard core point processes, where the points are forbidden to lie closer together than a certain minimum distance h. In our case, the inhibition distance h can be interpreted as the distance at which a potential emitter detects the emission from a neighbor. Below is the definition of the Matèrn point process.

Definition 2. *Let* Φ *be a homogeneous Poisson point process of intensity* λ*. We associate to each point* z *of* Φ*, a mark* m_z *uniformly distributed in* $[0, 1]$*. The points of the Matèrn point process are the points* z *of* Φ *such that the ball* $B(z, h)$ *centered at* z *and with radius* h *does not contain other points of* Φ *with marks smaller than* m_z*. Formally,*

$$\Phi_M = \{z \in \Phi s.t.m(z) < m(y) \forall y \in \Phi \cap B(z, h) \backslash \{z\}\}$$

This point process leads to more regular patterns as can be seen in Figure 1(c).

III. Link model

In this Section, we present a radio model based on the Poisson point process. The model is very general as it can be used for infrastructure-based wireless networks and ad hoc networks. It has been presented in [2] and [4]. The proofs and computations details can be found in these two articles.

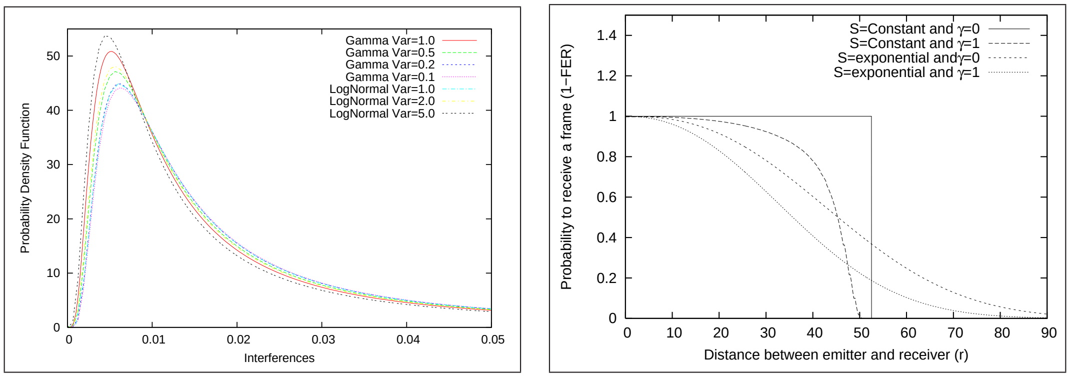
The model involves considering a homogeneous Poisson point process Φ of intensity λ to model emitter locations at a given time. In order to consider a particular link, we add two other points to the point process. The first one, located at the origin O, is the receiver. The second one y, located at a distance $\|y\|$ from the origin is the emitter. In the following, the different radio properties are related to the link between these two points.

A. Interference

Interference is one of the major quantities involved in the computations of link properties such *SINR*, *BER* and the Frame Error Rate (*FER*). Under certain assumptions, the interference at a given location can be considered as the sum of all of the signals from all of the emitters. Let $I_\Phi(x)$ be the interference at x. $I_\Phi(x)$ is defined as

$$I_\Phi(x) = \sum_{z \in \Phi} S_z l(\|x - z\|) \qquad (1)$$

where S_z is the emission power from z and $l(.)$ is the attenuation function. The sequence $(S_z)_{z \in \Phi}$ is



(a) *pdf* of interference.

(b) Probability of receiving a frame w.r.t. the distance.

Fig. 2. Interference and FER.

a sequence of positive random variables. It allows us to take into account radio phenomenae such as fading and shadowing. The probability density function of the interference cannot easily be computed. Nevertheless, due to the nature of the Poisson point process, we can compute its Laplace transform:

$$\begin{aligned} L_{I_\Phi(x)}(s) &= \mathbb{E}\left[e^{-sI_\Phi(x)}\right] \\ &= \mathbb{E}\left[\prod_{z\in\Phi} e^{-sS_z l(\|x-z\|)}\right] \end{aligned} \quad (2)$$

Under certain assumptions on S_z and $l(.)$ given in [4], the right hand side of equation 2 can be computed. We use the generating functional of the Poisson point process (see [25] for more details), leading to

$$L_{I_\Phi(x)}(s) = e^{-\lambda 2\pi \int_0^{+\infty}\left(1-\mathbb{E}\left[e^{-sSl(u)}\right]\right)u du} \quad (3)$$

where S is a random variable with the same distribution as the family (S_z). We can then use Laplace inversion techniques to obtain the probability density function (*pdf*) of the interference numerically. In Figure 2(a), we plotted the interference *pdf* for different radio environment. We considered S as the product of two random variables S_h and F, respectively modeling *shadowing* and *fading*. S_h follows a log-normal distribution whereas F follows a gamma distribution [24]. We observe that the distribution always presents the same form: a peak and a heavy tail. This observation confirms the results of [12], [17], [26], where a heavy-tailed interference distribution is observed for Poisson distributed interferers. Several distributions such as K-distribution, Weilbull, logNormal or Laplacian distributions have been proposed to model or extrapolate this heavy-tailed distribution. More recently, alpha-stable distributions have also been proposed [17].

These results are linked to the nature of the point process. For other point processes, such as the Matèrn, which models the CSMA/CA protocol, results may be very different. For low intensity, interference distributions are similar for the two point processes. But for higher intensity, there is a significant difference. The intensity of the Matèrn point process is bound by $\frac{1}{\pi h^2}$, limiting the interference level compared to the Poisson point process. Another difference is the tail of interference *pdf*, which is heavier for the Poisson point process as we can have several emitters very close to the points where we measure interference. For the Matèrn, the inhibition distance between the emitters limits the number of emitters in the neighborhood. The properties of interference based on Matèrn point processes have been studied in [23] and [1].

B. FER

The computation of the Frame Error Rate (*FER*), is often based on the *SINR*. We define the SINR for a transmission from y to the origin O as

$$SINR = \frac{Sl(\|y\|)}{W + \gamma I_\Phi(O)}$$

where W represents the noise. S is the signal power for an emission from y to O. It follows the same distribution as the sequence $(S_z)_{z\in\Phi}$. γ is a factor that allows to fine-tune the effect of transmission from the other emitters on the interference. There are different approaches for computing the probability of a frame not being received. For instance, we can consider that a frame is not received if the *SINR* is less than a given threshold θ:

$$FER = \mathbb{P}\left(SINR < \theta\right) \quad (4)$$

The quantity above cannot be analytically computed in most cases. But, it can be numerically evaluated from the distribution of interference obtained by the inversion of the Laplace transform. Nevertheless, there is a case where we can find a closed formula [4]. It corresponds to the case where S and $(S_z)_{z\in\Phi}$ follow exponential distributions (with parameter μ) modeling a $Rayleigh\ fading$. Under this particular assumption, equation 4 can be expressed with regard to the Laplace transform of Φ (given by equation 3):

$$\begin{aligned} FER &= \mathbb{P}\left(\frac{Sl(\|y\|)}{W+\gamma I_\Phi(O)} < \theta\right) \\ &= \mathbb{P}\left(S < \theta\frac{W+\gamma I_\Phi(O)}{l(\|y\|)}\right) \\ &= \mathbb{E}\left[e^{-\theta\frac{W+\gamma I_\Phi(O)}{l(\|y\|)}}\right] \\ &= L_{I_\Phi(O)}\left(\frac{\mu\theta\gamma}{l(\|y\|)}\right)L_W\left(\frac{\mu\theta}{l(\|y\|)}\right) \end{aligned}$$

In Figure 2(b), we plotted the probability that the receiver at the origin receives the frame (1-*FER*) when the distance between y and O varies ($\|y\| = r$ in the figures). We considered two distributions for S, the degenerate distribution ($S = constant$) and the exponential distribution. We also considered the cases where $\gamma = 0$ and where $\gamma = 1$. For $\gamma = 0$, interference is not taken into account. A frame is then received if $\mathbb{P}(SNR > \theta)$. The case $\gamma = 1$ supposes that all the emitters emit on the same channel. All the power transmissions from other emitters are then taken into account in the *SINR*. $S = constant$ and $\gamma = 0$, corresponds to the simplest case: $1 - FER = 1$ until $\frac{Sl(r)}{W}$ reaches θ, then $1 - FER = 0$. When $\gamma = 1$, the interference penalizes the value of the *SINR* and $1 - FER$ becomes smaller. When S is exponentially distributed, the curve decreases slowly from 1 to 0. The probability that the frame is lost may be not negligeable, even if the distance between the emitter and the receiver is small, while the probability of the frame being received is positive even when the distances are great. A more detailed presentation of these results can be found in [5].

IV. Connectivity

The study of connectivity is related to the study of a graph (E, V), where the vertex V are the nodes of the ad hoc network, and where the edges E are the links between the nodes. Except where otherwise stated, we suppose that node locations are represented by a Poisson point process Φ with intensity λ. We say that the newtork is connected or connex, if and only if there is a path between all the pairs $(x, y) \in V^2$, i.e. between all the pairs of nodes. A path between two nodes (x, y) is a finite sequence of nodes $(x_k)_{k=0,..,N}$ such that $x_0 = x$, $x_N = y$, and where the edges $(x_k, x_{k+1}) \in E$ for all $k = 0, .., N - 1$.

The radio model has an important impact on connectivity as it defines when there is an edge between two nodes. We consider here two kinds of radio models. A simplistic model, called the Boolean model, suppose that there is a link between two nodes if and only if the distance between the two nodes is less than a certain range R. The second model is the Signal To Interference Ratio Graph (*STIRG*) presented in [6]. It supposes that a directional link exists between two nodes (x, y) if the *SINR* at y is smaller than a given threshold θ:

$$\frac{Sl(\|x-y\|)}{W+\gamma I_\Phi(y)} < \theta \qquad (5)$$

where $I_\Phi(y)$ is the interference at y and S is constant and the same for all the nodes. The unidirectional links are ignored, therefore a link exists in the STIRG graph if both links (x, y) and (y, x) fulfill the condition given by equation 5.

In Figures 3(a), 3(b) and 3(c), we plotted the boolean model and *STIRG* graphs for the same sample of the Poisson point process and two different values of θ. It is clear that the STIRG model limits the connectivity as it takes into account interference generated by the other nodes.

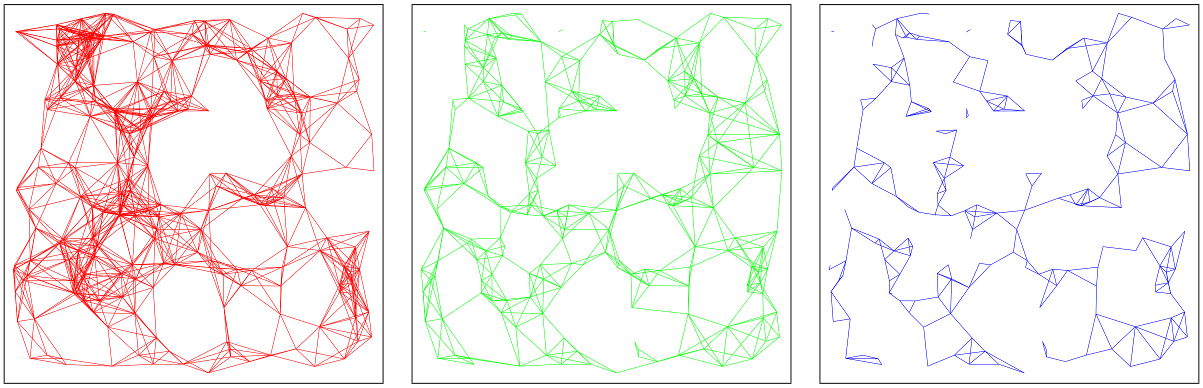
In the next two Sections, we distinguish two approaches: the case where the nodes are distributed in the whole plane, and the case where they are distributed in a finite area.

A. The infinite case

In the case where the point process is distributed in $\mathbb{R}^2$, it does not make sense to compute the probability of the graph being connex. Indeed, this probability is always nil as there is always a positive probability of a node being isolated. The study of connectivty is then related to the existence of an infinite component, i.e. a connex subgraph with an infinite number of nodes, rather than the full connectivity in the whole graph.

The study of such a component is linked to percolation theory. A reference book on percolation for the boolean model is [21]. For this model, it has been shown that there is a critical intensity λ_c (for a given range) for which the infinite component exists with a positive probability. When $\lambda < \lambda_c$, it is the subcritical phase where the infinite component exists with probability 0. In the supercritical phase (when $\lambda > \lambda_c$) the probability of the existence of an infinite component is strictly positive and, is unique when it exists. For discrete percolation (see [13]) the critical value has been analytically computed, but there are only numerical evaluations or analytical bound for the continuum case [18].

For the STIRG model, the authors of [6] tried to find similar results. In the STIRG model, when the intensity of the underlying Poisson point process increases, it does not improve the connectivity as it leads to a greater interference level. However, they show that if $\lambda > \lambda_c$, a critical value of γ_c exists for which the graph is subcritical (for $\gamma < \gamma_c$) and supercritical (for $\gamma > \gamma_c$). The existence of this critical value is discussed in [6], [7], [10], and depends on the attenuation function $l(.)$.



(a) Boolean graph. (b) STIRG with $\theta = 2.0$. (c) STIRG with $\theta = 5.0$.

Fig. 3. Connectivity graphs.

B. *The finite case*

In the case where the point process is disitributed in a finite area, the approach is different. The observation windows where the nodes are distributed are generally a ball or a square of unit area. n nodes are then uniformly distributed in this area.

For the boolean model, instead of the infinite case, the probability of having the graph fully connected is positive. But if the radio range is less than the diameter of the observation window, the probability of the graph not being connex is also positive whatever the value of n. Indeed, the probability of a node being isolated (with no link) is always positive. As a result, the connectivity for the finite case is also considered as an asymptotic property. It consists in studying the tradeoff between the radio range and the number of nodes as n tends to infinity to get a fully connected graph. The most important result is presented in [14]. The authors show that if the radio range $R(n)$ verifies $\pi R^2(n) = \frac{log(n)+c(n)}{n}$ with $\lim_{n\to+\infty} c(n) = +\infty$, then the graph is asymptotically connex ($\lim_{n\to+\infty} \mathbb{P}\left(\textit{the graph is fully connected}\right) = 1$).

For the STIRG model, it has been proposed in [6] to transpose the results of the infinite graph to the finite one.

V. Capacity

Capacity plays an important role in the performance evaluation of ad hoc networks as it limits the applications which can be used, the number of nodes for a given application, etc. The very famous paper which addressed the problem of capacity for the first time is [15]. Numerous papers followed with more elaborate radio assumptions, but the definition of capacity and point processes used to model node locations still remain the same. The observation window is a ball of unit area, denoted B. We consider n nodes distributed uniformly in B. We suppose that each node is a source emitting to another node randomly chosen among the other nodes. The capacity, often called throughput per node or feasible throughput, and denoted $c(n)$, is then defined as the mean number of bits per second that a node is able to transmit to its destination. This capacity must be obtained for all the pairs (source-receiver).

The model used to set up the links between the nodes is similar to the STIRG model. We shall assume that there is a link between two nodes if the *SINR* is greater than a given thershold θ. The first major result on capacity, given in [15], states that the capacity is $O\left(\frac{1}{\sqrt{n}}\right)$ and $\Omega\left(\frac{1}{\sqrt{n \log n}}\right)$.

$c(n) = \Omega\left(\frac{1}{\sqrt{n \log n}}\right)$ if there is a constant a such that

$$\lim_{n\to+\infty} \mathbb{P}\left(c(n) \geq \frac{1}{\sqrt{n \log n}}\right) = 1 \qquad (6)$$

This result gives a lower bound on the achievable throughput. The other result, $c(n) = O\left(\frac{1}{\sqrt{n}}\right)$, is similar to formula 6, but gives an upper bound on the capacity. To prove the lower bound, the authors built a routing mechanism and a TDMA scheme, which allows each node to reach this capacity. For the upper bound, they showed that an inhibition radius exists around each emitter. A transmission in this area from another emitter will make the reception impossible. Therefore, it bounds the number of simultaneous emitters and thus the capacity.

A natural question arises. Is the upper bound given by Gupta et. al achievable or not? In [11], the authors showed that, for a particular attenuation function, the upper bound is also a lower bound $\left(c(n) = \Omega\left(\frac{1}{\sqrt{n}}\right)\right)$. As above, they prove it by building a particular routing scheme. A certain number of ”highways”, made up of connected sets of nodes and crossing the network horizontally, are used to transport all traffic from all of the sources. These highways are then the bottleneck of the network. Each highway is built in such a way that the capacity of each highway is constant, and is responsible for carrying the traffic of $\sqrt{n}$ nodes. This construction leads to a feasible throughput proportional to $\frac{1}{\sqrt{n}}$.

It is worth noting that all these results hold for a particular family of attenuation function $l(u) = u^{-\alpha}e^{-au}$ with $\alpha > 2$ and $a \geq 0$. These functions tends to infinity as the distance u tends to 0. As a consequence, there is always a distance between the source and the receiver for which the transmission will succeed whatever the interference level. Moreover, close interferers can drastically increase interference. So, depending on the point process intensity, this attenuation function increases or decreases the connectivity compare with a more realistic attenuation function with the form $l(u) = min\left(1, x^{-\alpha}e^{-ax}\right)$. The effects of the attenuation function on capacity is discussed in [10].

In these radio models, the link exists if the *SINR* is greater than θ. The transmissions on these links are then supposed to be systematically successful. For a more realistic radio model, given that, whatever the value of the *SINR*, transmission errors are always possible, the results on capacity change drastically. In [22], the authors show that this assumption decreases the capacity. They found an upper bound on the capacity proportional to $\frac{1}{n}$ $\left(c(n) = O\left(\frac{1}{n}\right)\right)$. For other ad hoc networks, such as Vehicular Ad-hoc NETworks (*VANET*), where the nodes are vehicles moving on a road or a highway, the previous models cannot be used. Indeed, the toplogy is more a set of nodes distributed on a straight line than in a two-dimensional area. These chains of nodes are particularly penalizing for capacity, which is found proportional to $\frac{1}{n}$ (see [19]).

VI. Conclusion

We have presented a brief overview of spatial models for the performance evaluation of ad hoc networks. The use of the point processes allows us to estimate the performances of these networks. Spatial modeling proved to be a powerful tool for modeling ad hoc networks. It allows us to understand the effects of the different parameters on performance, and to observe the behavior of these networks on different scales.

In addition to link properties such as interference, *SINR* and *FER*, we have presented results on connectivity and capacity. All analytical results of these aspects are asymptotic results. They give qualitative results on the behavior of ad hoc networks, and can be used as approximations for their dimensioning. But most of the existing models cannot be used as fine grain dimensioning tools. Additional work is still necessary to take into account more realistic radio assumptions and point processes.

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